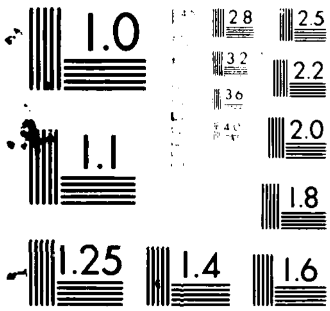
AD-A109 578 SYSTEMS CONTROL INC (VT) PALO ALTO CA F/G 17/7
MULTITARGET TRACKING STUDIES.(U)
JUL 81 B FRIEDLANDER N00014-79-C-0743
UNCLASSIFIED SCI-5334-02 NL

1 of 1
AD
A109578

END
DATE
FILMED
2 82
DTIC



MICROCOPY RESOLUTION TEST CHART

LEVEL

SYSTEMS CONTROL TECHNOLOGY, INC.

1801 PAGE MILL RD. □ P.O. BOX 10180 □ PALO ALTO, CALIFORNIA 94303 □ (415) 494-2233

Report No. 5334-02 July 1981

MULTITARGET TRACKING STUDIES

Phase II Final Report

Prepared by:

B. Friedlander
Systems Control Technology, Inc.

Prepared for:

James G. Smith
Statistics and Probability Program (Code 411SP)
Mathematical and Information Sciences Division
Office of Naval Research

Under Contract:

N00014-79-C-0743
NR 277-287

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 5334-02	2. GOVT ACCESSION NO.: AD-A109 578
	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): Multitarget Tracking Studies: Phase II Final Report	5. TYPE OF REPORT & PERIOD COVERED: Final Report 6/4/80 - 7/3/81
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Benjamin Friedlander	8. CONTRACT OR GRANT NUMBER(s): N00014-79-C-0743
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Systems Control Technology, Inc. 1801 Page Mill Road Palo Alto, California 94304	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 65153N RO-1457W NR277-287
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research Arlington, Virginia 22217	12. REPORT DATE: July 1981
	13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

autoregressive moving-average
recursive maximum likelihood
tracking
adaptive signal processing
time difference of arrival

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents the results of Phase II of an investigation of a new concept for tracking multiple targets. This concept is based on parametric modeling of the observed data. The recursive maximum likelihood algorithm was used to estimate the parameters of an ARMA model. The single channel version of the algorithm was used as an adaptive IIR filter. The algorithm was applied to: adaptive line enhancement, adaptive noise cancelling, high resolution spectral estimation, time delay estimation, and adaptive deconvolution. The multichannel version of

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. (continued)
the algorithm was used to estimate TDOAs and spectral parameters of multiple targets. Results based on simulations using synthetic data are presented.

TABLE OF CONTENTS

A

1. INTRODUCTION AND SUMMARY

The objective of the Multitarget Tracking Studies (MTS) project is to develop and evaluate a parameter modeling concept for tracking multiple targets. This report summarizes the accomplishments on phase II of the project, during the period of June 1980 to June 1981. The results obtained during phase I of MTS were reported in [1].

The MTS concept is based on modeling the observed data as a multichannel autoregressive moving-average (ARMA) process. The parameters of the model provide a compact representation of target parameters such as spectrum and time-difference-of-arrival (TDOA). These parameters can be used as inputs to a tracking algorithm, a target classification program, etc. The unique feature of the MTS technique is the simultaneous estimation of multiple target parameters. Most conventional processing techniques estimate parameters of one target at a time, considering the other targets as sources of interference.

Parametric modeling of ARMA signals at low signal-to-noise ratios presents a difficult and challenging problem, even in the single channel case. It was clear from the start that single-input single-output ARMA models have to be studied before tackling the multi-input multi-output models that arise in the multitarget case. Accordingly, most of the effort in phase I and part of the effort in phase II was devoted to the single channel case. The emphasis was on developing a recursive parameter estimation algorithm capable of working at the low-signal-to-noise ratios that are typically encountered in the undersea surveillance scenario (e.g. -10 to -15 dB). In section 2 we present the single channel algorithm which is a more robust version of the one presented in [1]. This algorithm is used as an adaptive Infinite Impulse Response (IIR) filter to perform a number of functions such as: adaptive line enhancement, adaptive noise cancelling, adaptive TDOA estimation, adaptive deconvolution and high resolution spectral estimation. Adaptive filters of this kind are useful for line tracking, own ship noise reduction, inter-array processing and other surveillance applications. The IIR adaptive filter has improved performance compared to conventional Finite Impulse

filter, especially under low signal-to-noise conditions, as demonstrated by the results in [2]-[4].

In section 3 we describe the multichannel ARMA modeling algorithm developed for the multitarget case. The algorithm was tested under high signal-to-noise conditions and was shown to be able to estimate correctly the TDOA's and spectral parameters of two targets. However, a number of theoretical as well as practical issues remain to be resolved before the performance of the algorithm can be adequately assessed. This will be done as part of phase III of the MTS project in which a more robust version of the multitarget processor will be developed. The main difficulties encountered in the ARMA approach are related to the question of structural constraints. The parametric tracking model (comprised of a spectral model and a propagation model) considered in our work leads to a certain linear transfer function matrix describing the signals received by the sensors. The transfer matrix is called the Right Matrix Fraction Description (RMFD) of the system. The RMFD associated with the tracking model has a very special structure as will be discussed in section 3. Estimating the parameters of this matrix involves the solution of a highly nonlinear optimization problem. The ARMA model provides an alternative parameterization (a Left Matrix Fraction Description) which is much easier to compute. Unfortunately, the special structure of the RMFD is not shared by the ARMA model. The algorithm we currently use estimates the parameters of a general ARMA model with no structural constraints. Because of this, the tracking model is highly overparametrized, leading to uniqueness problems and to degraded performance at low SNR. To make the MTS processor more robust it seems that we will have to replace the recursive parameter estimation algorithm originally used with a different type of algorithm capable of enforcing the special model structure. This will probably lead to a more complex algorithm structure.

The research performed so far on the MTS project resulted in a number of publications, as listed in section 5. In this report we present a brief summary of this research, deferring many details to these publications. Two of the key papers, one for the single channel case and one for the multitarget case, are included as appendices A and B. Further details on the MTS algorithms are presented in appendices C and D.

2. SINGLE TARGET ALGORITHMS

2.1 THE RML ALGORITHM

The MTS processor is based on fitting an ARMA model to an observed time-series. Several recursive parameter estimation algorithms were studied in phase I for performing this model fitting. The recursive maximum likelihood algorithm (RML) was chosen as the one most suitable for our problem. The RML algorithm estimates the parameters of an ARMA model of the type

$$y_t = -\sum_{i=1}^{NA} a_i \, y_{t-i} + \sum_{i=1}^{NB} b_i \, u_{t-i} + \sum_{i=1}^{NC} c_i \, v_{t-i} + v_t \qquad (1)$$

where $\{y_t, u_t\}$ are the data sequence and v_t is an unobservable white noise sequence. The RML algorithm is specified by the following set of equations (see Appendix A)

$$\Theta = [a_1, \ldots, a_{NA}, b_1, \ldots, b_{NB}, c_1, \ldots, c_{NC}]^T = \text{parameter vector} \qquad (2a)$$

$$\phi_t = [-y_{t-1}, \ldots, -y_{t-NA}, u_{t-1}, \ldots, u_{t-NB}, e_{t-1}, \ldots, e_{t-NC}]^T = \begin{array}{l}\text{data}\\ \text{vector}\end{array} \qquad (2b)$$

$$\psi_t = [-\tilde{y}_{t-1}, \ldots, -\tilde{y}_{t-NA}, \tilde{u}_{t-1}, \ldots, \tilde{u}_{t-NB}, \tilde{e}_{t-1}, \ldots, \tilde{e}_{t-NC}]^T = \begin{array}{l}\text{filtered}\\ \text{data vector}\end{array} \qquad (2c)$$

$$P_t = [P_{t-1} - P_{t-1}\psi_t \, \psi_t^T \, P_{t-1}/(\lambda_t + \psi_t^T \, P_{t-1}\psi_t)]/\lambda_t \qquad (3)$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} + P_t \, \psi_t \, (y_t - \phi_t^T \, \hat{\theta}_{t-1}) \qquad (4)$$

$$e_t = y_t - \phi_t^T \, \hat{\theta}_t \qquad (5)$$

$$\tilde{y}_t = (1/D_t(z)) \, y_t, \; \tilde{u}_t = (1/D_t(z)) \, y_t, \; \tilde{e}_t = (1/D_t(z)) \, e_t \qquad (6)$$

The pre-filter $D_t(z)$ is usually taken to be

$$D_t(z) = 1 + \hat{c}_1(t) \, z^{-1} + \ldots + \hat{c}_{NC}(t) \, z^{-NC} \qquad (7)$$

The expontential forgetting factor is usually chosen as,

$$\lambda_t = \lambda \lambda_{t-1} + (1-\lambda),$$

and the initial conditions are

$$\hat{\theta}_0 = 0, P_0 = \sigma I.$$

Several features were added to the algorithm to guarantee its convergence and make it more robust:

(i) Stability monitoring

The stability of the pre-filter $D_t(z)$ needs to be tested. Whenever $D_t(z)$ becomes unstable the parameter estimates $\hat{\theta}_t$ need to be projected back into a region of stability [5].This can be done by setting the parameters back to their value before the last update: $\hat{\theta}_t = \hat{\theta}_{t-1}$. Another procedure is to go back only part of the way, e.g.,

$$\hat{\theta}_t = \hat{\theta}_{t-1} - sV_t \tag{8a}$$

where

$$V_t = P_{t-1} \phi_t (y_t - \phi_t^T \hat{\theta}_{t-1}) \tag{8b}$$

The coefficient s is set initially to max $\{1/2, 10^{-6}/|V_t|\}$, where $|V_t|$ denotes the sum of the absolute values of the entries of the vector V_t. If the new parameter estimates Θ_t are still unstable we set $s = \max \{s/2, 10^{-6}/|V_t|\}$ and repeat the process. The stability test algorithm is given by the following program:

```
      FUNCTION NSTAB(T,NT)
C     PERFORMS A TEST WHETHER THE POLYNOMIAL T HAS ALL ROOTS
C     INSIDE THE UNIT CIRCLE OR NOT
C     T = 1+T(1)*Q**(-1)+...+T(NT)*Q**(-NT)
C     WHERE Q**(-1) IS THE BACKWARD SHIFT OPERATOR
C
C     T IS A VECTOR CONTAINING THE COEFFICIENTS OF THE
C     POLYNOMIAL
C     NT IS THE ORDER OF THE POLYNOMIAL /MAX=30/
C
C     THE FUNCTION IS RETURNED 0 IF ALL ROOTS ARE INSIDE THE
C     UNIT CIRCLE, I.E. FOR A STABLE POLYNOMIAL; OTHERWISE AS 1
      DIMENSION T(1)
      DIMENSION TT(30), TTT(30)
C
      NSTAB=0
C
      DO 10 I=1, NT
   10 TT(I)=T(I)
C
      DO 50 I=NT,1,-1
      AK=TT(I)
      IF(ABS(AK).GE.1.) GO TO 60
      T1=1.-AK*AK
      I1=I-1
      IF(I1) 50,50,20
   20 DO 30 J=1,I1
   30 TTT(J)=(TT(J)-AK*TT(I-J))/T1
      DO 40 J=1,I1
      40 TT(J) = TTT(J)
   50 CONTINUE
C
      GO TO 70
   60 NSTAB=1
   70 RETURN
      END
```

(ii) Modified Pre-filtering

The choice of the pre-filter $D_t(z)$ has an effect both on the asymptotic properties of the RML algorithm and on its transient behavior. The choice indicated in equation (7) guarantees asymptotic convergence and asymptatic efficiency. However, other choices seem to have faster convergence rate, which is an important factor in adaptive processing applications.

We experimented with a modified pre-filter of the form:

$$D_t(z) = 1 + k\hat{c}_1(t)z^{-1}+\ldots+k^{NC}\,\hat{c}_{NC}(t)z^{-NC} \tag{9}$$

where $0<k<1$. For different choices of the parameter k we get algorithms with different types of behavior. In particular, k=1 gives the RML algorithm while k=0 gives the extended least-squares algorithm. When using the algorithm on narrowband signals in noise we found that using intermediate values of k often improved the performance of the algorithm. A more detailed discussion of the modified pre-filter is given in [6].

(iii) The Square-Root Form of the RML

The basic form of the RML involves the updating of the covariance matrix P_t using the difference Eq.(4). This type of equation does not guarantee the positive definiteness of P_t and can lead to numerical problems. It is well known that a much better behaved solution to least-squares estimation problems can be obtained by square-root techniques which update $P_t^{1/2}$ rather than P_t (where $P_t^{1/2}P_t^{T/2} = P_t$) [7],[8]. We used the following algorithm to implement a square-root version of the RML: Let

$$P_{t-1} = \tilde{U}\,\tilde{D}\,\tilde{U}^T = \text{old covariance matrix}$$

$$P_t = \hat{U}\,\hat{D}\,\hat{U}^T = \text{new covariance matrix}$$

where $\tilde{U}$, $\hat{U}$ are upper triangular with unity on the diagonal, and $\tilde{D}$, $\hat{D}$ are diagonal matrices. Then the following are the update equations for the U-D factors:

- Initialization:

$$\tilde{U} = I, \ \tilde{D} = \text{diag}\ \{\sigma\}$$

- First step:

$$f = \tilde{U}^T \psi_t, \quad \text{where} \quad f^T = [f_1, \ldots, f_n]$$

$$v = \tilde{D}f, \text{ i.e., } v_i = \tilde{d}_i f_i \text{ for } i=1, \ldots, n$$

$$\hat{d}_1 = \tilde{d}_1 / \alpha_1 \quad \text{where} \quad \alpha_1 = \lambda_t + v_1 f_1$$

$$\bar{k}_1^T = [v_1, 0, \ldots, 0]$$

- Main Loop

for $j = 2, \ldots, n$ do

$$\alpha_j = \alpha_{j-1} + v_j f_j$$

$$\hat{d}_j = \tilde{d}_j \ \alpha_{j-1}/(\alpha_j \ \lambda_t)$$

$$\hat{u}_j = \tilde{u}_j + a_j \ \bar{k}_{j-1}, \quad \text{where} \quad a_j = -f_j/\alpha_{j-1}$$

$$\bar{k}_j = \bar{k}_{j-1} + v_j \ \tilde{u}_j$$

We use the notation $\tilde{u}_i$, $\hat{u}_i$ for the columns of $\tilde{U}$, $\hat{U}$, i.e.,

$$\tilde{U} = [\tilde{u}_1, \ldots, \tilde{u}_n], \ \hat{U} = [\hat{u}_1, \ldots, \hat{u}_n]$$

- Compute New Gain

$$K_t = \bar{k}_n/\alpha_n$$

- Update Parameter Vector

$$\hat{\theta}_t = \hat{\theta}_{t-1} + K_t (y_t - \phi_t^T \hat{\theta}_{t-1})$$

- Compute Residual

$$e_t = y_t - \phi_t^T \hat{\theta}_t$$

An implementation of these update equations can be found in [8, pp. 100-101]. A slight modification was needed to account for an extra division by λ_t, which appears in (4), but not in the standard Kalman update of the covariance matrix P_t.

The square-root RML has shown somewhat improved performance, especially at low SNR, and we are now using only this version of the algorithm. Since our simulations were performed using high precision floating-point computuaions, the difference between the regular and square-root normalized RML is not large. A much more pronounced differnce is expected to occur when using fixed point finite-word length computations.

The RML algorithm with the various featrues described above is quite robust and has been used successfully on narrowband data at SNR's as low as -15 dB. The algorithm will probably work at lower SNR given sufficient data. In our simulations we used not more that 4096 data points per run.

2.2 APPLICATIONS OF THE RML FOR ADAPTIVE PROCESSING

The RML algorithm can be used for a variety of adaptive processing applications. In appendix A we present a summary of our work on using the RML for adaptive line enhancement, adaptive noise cancelling, adaptive time delay estimation, adaptive deconvolution and high resolution spectral estimation. Here we provide brief descriptions of these applications and give references to project publications containing more details.

(i) Adaptive Line Enhancement [2]

The RML algorithm can be used as an adaptive prediction filter for ARMA processes. The predicted value $\hat{y}_{t|t-1}$ is given by

$$\hat{y}_{t|t-1} \cong -\sum_{i=1}^{NA} \hat{a}_i(t-1)y_{t-i} + \sum_{i=1}^{NC} \hat{c}_i(t-1)\varepsilon_{t-i} \quad , \tag{10a}$$

where

$$\varepsilon_{t-i} = y_{t-1} - \phi_{t-1}^{T}\hat{\theta}_{t-i-1} \; .$$

The structure of the IIR filter is depicted in Fig. 1. The coefficients of this filter are adjusted at each time step according to the update equation (2) - (7) of the RML algorithm. When the input is a narrowband signal corrupted by white noise the adaptive predictor will form a bandpass filter around the spectral lines and reject much of the noise. In other words, the adaptive predictor is precisely the so-called adaptive line enhancer (ALE) [9].

The ALE's currently used by the Navy and other users of adaptive signal processing are all based on FIR filtering. As discussed in [2], the IIR filter has a potential for performance improvement over FIR filters, especially at low SNR's. The SNR improvement of the FIR-ALE is proportional to the filter order. Thus, to achieve significant noise rejection, filters of very high order have to be used, leading to fairly complex hardware. Filters with 4096 weights and more are not uncommon. The IIR-ALE can achieve better performance with a very low order filter, the order being just twice the number of spectral lines of interest. This can lead to significant computational saving and simpler hardware. We have also developed a lattice version of the IIR-ALE which is particularly attractive for special purpose hardware implementation [10].

(ii) Adaptive Noise Cancelling [3]

The noise cancelling technique makes use of an auxiliary or reference input which provides some information about the noise component that is corrupting the signal. From this "side information" an estimate of the noise is obtained and then subtracted from the primary input to "cancel out" the

additive noise component. The reference input may be an extra beam steered in the direction of known noise sources (e.g. own ship) or extra sensors in a towed array giving additional information about flow noise. These noise subtraction techniques are very sensitive to modeling errors since they are

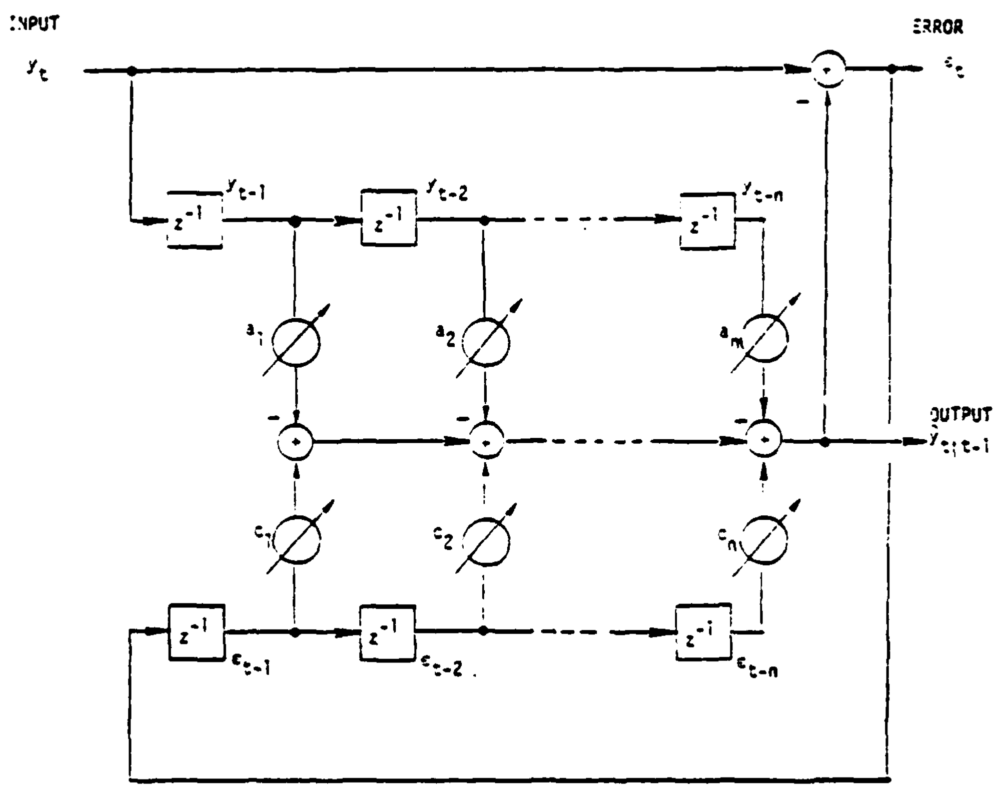


Figure 1. The IIR Adaptive Line Enhancer.

essentially equivalent to "bridge balancing". Therefore, noise cancelling is typically performed by an adaptive processor which properly adjusts the filter parameters.

All of the adaptive noise cancellers (ANC) currently in use seem to be of the FIR type. Using the RML algorithm we were able to develop an IIR-ANC whose structure is depicted in Fig. 2. In [3] it was shown that the general ANC problem can be solved by fitting the following model to the observed data at the primary (y_t) and reference inputs (u_t):

$$y_t = \frac{B(z)}{F(z)} u_t + \frac{C(z)}{D(z)} e_t \tag{11}$$

where e_t is an unmeasurable white noise process. A modified version of the RML algorithm [11] recursively estimates the parameters of this model, which are then used to adjust the filter coefficients.

Note that the IIR-ANC involves only the parameters of B(z) and F(z). It turns out that the remaining parameters C(z), D(z) can be used to form an adaptive line enhancer for the signal at the ANC ouput. In other words the RML algorithm can adjust simultaneously an IIR-ANC and an IIR-ALE, as depicted in Fig. 3. The combined ALE-ANC was tested on synthetic data with promising results, as reported in [3].

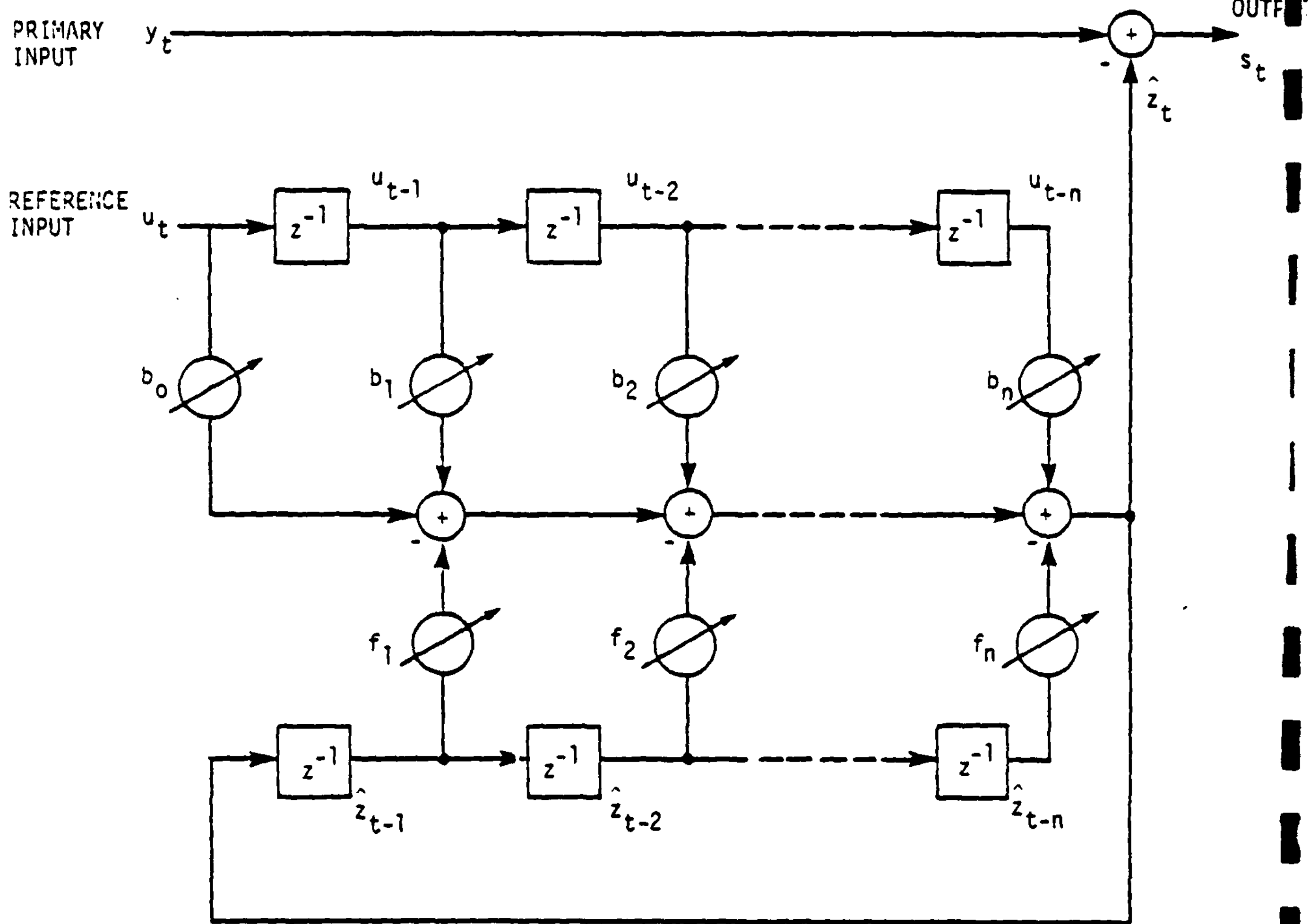


Figure 2. The IIR adaptive noise canceller.

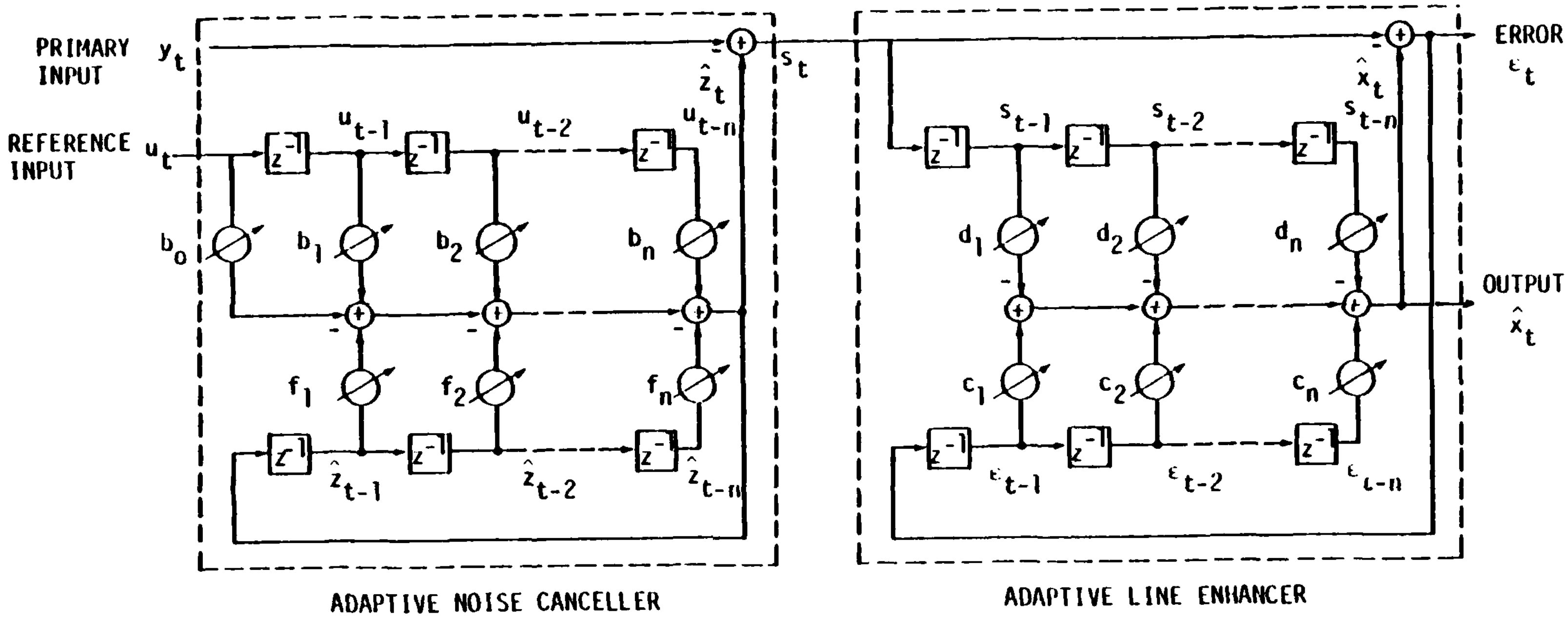


Figure 3. The combined ALE-ANC.

(iii) High Resolution Spectral Estimation [4]

Spectral estimation techniques play an important role in dectection, localization and classification of targets. High resolution AR and ARMA methods were used in many applications such as seismic data processing, radar and sonar processing and general time-series analysis problems. Several good techniques are currently available for AR modeling of time series. The ARMA modeling problem is much more difficult, and leads to optimization of a highly nonlinear likelihood function. Because of the computational complexity of the exact maximum likelihood technique, ARMA modeling is often avoided in practice in spite of its potential for improved resolution and better spectral matching. Various suboptimal batch processing techniques were proposed in the past for ARMA spectral estimation. The RML algorithm provides a suboptimal recursive solution technique which is well suited for adaptive processing applications. We tested the RML spectral estimator on several types of signals under various SNR conditions. Results have been good for sufficiently long data records, as reported in [4]. On short data records recursive techniques are bound to be inferior to batch techniques, because of the transient behavior of such algorithms. The RML can also be used in a batch mode, in which case it is passed several times over a given data record. When used in this mode the RML spectral estimates can match those of competitive ARMA techniques.

Signal processing problems often involve sinusoidal signals corrupted by additive noise. When it is known a priori that the signals of interest are sinusoids, this information can be used to enhance the performance of spectral estimators. When AR techniques are used,the presence of sinusoids means that the roots of the predictor A(z) lie on the unit circle. Another interpretation of this property is that A(z) is a linear phase filter. Forcing the AR prediction filter to have linear phase characteristics leads to improved spectral estimation performance as was shown by Marple [12]. In [13] we developed several recursive least-squares algorithms for AR modeling which can be used for spectral estimation and other applications.

(iv) Adaptive Deconvolution

The need to extract a signal from a filtered version of that signal arises in many situations such as channel equalization, seismic data processing, speech analysis, and removal of shallow water reflections in an active sonar system. The RML algorithm can be directly applied to these problems provided that the signal (before it was filtered) is sufficiently white. In this case the RML algorithm is used to implement an adaptive "whitener". The prediction error sequence will then provide an estimate of the original signal. Note that this technique can handle signals that have passed a pole-zero filter, whereas most of the current deconvolution techniques are limited to all-pole filters. Some examples of adaptive deconvolution using the RML algorithm can be found in appendix A. In speech analysis and other applications it is known a priori that the signal is a sequence of impulses of random amplitudes occuring at random points in time. This knowledge can be used to improve the performance of the deconvolution technique by performing a nonlinear opertion in the feedback loop within the RML algorithm; see [14] for details.

(v) Adaptive Time Delay Estimation

Delay estimation is an important technique for target localization by multiple sensors or multiple arrays. Many different techniques for delay estimation have been proposed, as can be seen in [15]. These techniques involve pre-filtering followed by cross correlation and are usually performed in the frequency domain. The RML provides several options for adaptive delay estimation in the time domain. In one configuration the RML algorithm is used as an adaptive whitening filter. The outputs of two sensors are whitened and then cross-correlated. In another configuration, the RML algorithm is used to fit an MA model relating the output of one sensor to the output of another. This idea is similar to the one proposed in [16]. Both of these techniques seem to work quite well as reported in [1]. However, a more interesting possibility is to use a multichannel version of the RML, as will be discussed in section 3.

2.3 PERFORMANCE EVALUATION

The single channel RML algorithm was tested in the context of several specific applications, as described above. Based on our experience so far the RML appears to be a viable technique for adaptive IIR filtering capable of operating at low SNR's. The RML algorithm provides asymptotically efficient parameter estimates, i.e., the variance of the parameter estimate $\hat{\phi}$ approaches the Cramer-Rao lower bound [17]. The asymptotic performance bound of the algorithm can be evaluated using the results presented in [18]. The algorithm is guaranteed to converge when the signal y_t is an ARMA process, provided that the orders NA and NC are at least as high as the true model order [19], [20]. The SNR gain of the IIR-ALE was evaluated analytically in [2]. The performance of the RML in other applications was evaluated mainly by simulation studies [3], [4], [21] in which it compared favorably with currently used adaptive IIR filters and AR estimation techniques such as the Maximum Entropy Method.

The main difficulty encountered with the RML algorithm is its fairly long transient phase which typically lasts for several hundred data points. In signal processing applications involving continuous filtering of data, the transient phase is of little importance. Once the algorithm "locks-on" the right parameter estimates it will continue tracking. In applications involving short data records the convergence rate of the RML may not be sufficiently fast. In general we found that the convergence of the RML was accelerated as the parameter k in the modified pre-filter was made smaller. This is in agreement with observations by other researchers that the Extended Least-Squares technique (k=0) is usually faster than the RML algorithm (k=1). It is possible that further modifications can be made to increase the convergence rate of the algorithm. Unfortunately, no analytical results are available on the convergence rate of recursive stochastic algorithms of this type. It will be necessary to develop appropriate analysis tools before this issue can be properly addressed. However, the performance of the RML in its pesent form is probably good enough for most adaptive processing applications.

Finally we note that the RML algorithm was tested mainly on stationary data. Its capability for tracking signals with time-varying statistics was not evaluated adequately due to time and budget limitations.

3. THE MULTITARGET ALGORITHM

The MTS concept is based on fitting a multi-input multi-output parametric model to the observed data. In [1] and in appendix B we show that the following transfer function matrix arises from the physical model:

$$H(z) = N(z)D(z)^{-1} \tag{12}$$

where

$D(z) = \text{diag}\ \{1/d_i(z)\}$ = spectral model

$N(z) = [z^{-\tau_{ij}}]$ = propagation model

(τ_{ij} = propagation delay from target i to sensor j)

The outputs of the sensors Y(z) can be written as

$$Y(z) = H(z)U(z) + V(z) \tag{13}$$

where U(z), V(z) are independent white-noise processes. Using this problem formulation we need a way of estimating the parameters of the parametric model H(z) from observations of the sensor data sequence. Unfortunately, the parameters of the Right Matrix Fraction Description (RMFD) $H(z) = N(z)D(z)^{-1}$ are hard to estimate. It is much easier to estimate the parameters of a left Matrix Fraction Description (LMFD)$H(z) = A^{-1}(z)\ B(z)$.The RMFD arises naturally from the physical model and the target parameters appear in it directly. Furthermore, the RMFD has a very special structure (D(z) a diagonal matrix and N(z) a delay matrix) while no such structure is apparent in the LMFD. However, because of the much greater complexity of estimating RMFD parameters, we have concentrated so far on the LMFD.

In section 3.1 we present an extension of the RML algorithm for estimating the parameters of a LMFD of the transfer function H(z). In section 3.2 we present some simulation results using this algorithm. The problem of uniqueness and other theoretical issues are discussed in appendix B.

3.1 THE MULTICHANNEL RML

The RML algorithm described in section 2 can be extended to estimate the parameters of multichannel ARMA models of the form

$$y_t = -\sum_{i=1}^{NA} A_i \, y_{t-i} + \sum_{i=1}^{NB} B_i \, v_{t-i} + v_t \tag{14}$$

where y, v, are $p \times 1$ vectors and A_i, B_i are $p \times p$ matrices, p being the number of sensors. The components of v_t are uncorrelated white noise processes. Eq. (14) can be written as

$$y_t = \theta^T \bar{\phi}_t + v_t \tag{15}$$

where

$$\theta^T = [A_1, \ldots A_{NA}, B_1, \ldots, B_{NB}] \,, \qquad p \times (NA + NB)p \text{ matrix}$$

$$\bar{\phi}_t = [-y^T_{t-1}, \ldots, -y^T_{t-NA}, v^T_{t-1}, \ldots v^T_{t-NB}]^T, \qquad (NA + NB)p \times 1 \text{ vector}$$

or it can be decoupled into p equations of the type

$$y^j_t = \phi^T_t \, \theta^j + v^j_t, \qquad j = 1, \ldots, p \tag{16}$$

where θ^j is the j-th column of θ. For each of these equations we can now write down the single channel algorithm described earlier. All of these algorithms will have a common gain vector $P_t \, \psi_t$. Let

$$\phi_t = [-y^T_{t-1}, \ldots, -y^T_{t-NA}, e^T_{t-1}, \ldots, e^T_{t-NB}]^T \tag{17a}$$

$$\psi_t = [-\tilde{y}^T_{t-1}, \ldots, -\tilde{y}^T_{t-NA}, \tilde{e}^T_{t-1}, \ldots, \tilde{e}^T_{t-NB}]^T \tag{17b}$$

where e_t, $\tilde{y}_t$, $\tilde{e}_t$ are to be defined. The update equations are then given by

$$P_t = [P_{t-1} - P_{t-1} \, \psi_t \, \psi^T_t \, P_{t-1}/(\lambda_t + \psi^T_t \, P_t \, \psi_t)]/\lambda_t \tag{18}$$

$$\hat{\theta}^j_t = \hat{\theta}^j_{t-1} + P_t \, \psi_t (y_t - \phi^T_t \, \hat{\theta}^j_{t-1}) \tag{19}$$

$$e_t^j = y_t - \phi_t^T \hat{\theta}_t^j \ , \quad e_t = [e_t^1, \ldots, e_t^p]^T \tag{20}$$

$$\tilde{y}_t = D_t^{-1}(z)\ y_t \ , \quad \tilde{e}_t = D_t^{-1}(z)\ e_t \tag{21}$$

$$D_t(z) = I + k\hat{B}_1(t)z^{-1} + \ldots + K^{NB}\hat{B}_{NB}(t)z^{-NB} \tag{22}$$

The RML algorighm described above estimates the coefficients of the prediction filter

$$\varepsilon_t = y_t + \sum_{i=1}^{NA} \hat{A}_i(t-1)\ y_{t-i} \sum_{i=1}^{NB} \hat{B}_i\ (t-1)\ \varepsilon_{t-i} \tag{23}$$

where

$$\varepsilon_t = [\varepsilon_t^1, \ldots, \varepsilon_t]^T,$$

$$\varepsilon_t^j = y_t - \phi_t^T \hat{\theta}_{t-1}^j \ .$$

The prediction error sequence is white, but there is no guarantee that the components ε_t^j are uncorrelated. Since the original model (14) involved a vector with uncorrelated components, we need to decorrelate the components of ε_t as well. This can be done by pre-multiplying with $R_t^{-\varepsilon/2}$, where $R_t^{\varepsilon/2}$ is the upper triangular square root of the prediction error covariance matrix. In other words, the estimated model will be

$$\hat{H}(z) = \hat{A}_t^{-1}(z)\ \hat{B}_t(z)\ R_t^{-\varepsilon/2} \tag{24}$$

The square-root covariance matrix $R_t^{\varepsilon/2}$ can be computed recursively using the procedure described next. First note the following fact:

$$R_t^{\varepsilon} = (1-w_t)\ R_{t-1}^{\varepsilon} + w_t\ \varepsilon_t \varepsilon_t^T \ , \qquad R_o^{\varepsilon} = \varepsilon_o \varepsilon_o^T \tag{25a}$$

$$w_t = w_{t-1}/(w_{t-1} + \lambda_t) \ , \qquad w_o = 1 \tag{25b}$$

The parameter w_t is needed to make R_t^{ε} an unbiased estimator of $E\{\varepsilon_t \varepsilon_t^T\}$. If we were to compute

$$R_t^{\varepsilon} = \lambda_t R_{t-1}^{\varepsilon} + \varepsilon_t \varepsilon_t^T \tag{26}$$

then R_t^{ε} will be a scaled version of the true covariance, where the scaling factor is η_t

$$\eta_t = \lambda_t \eta_{t-1} + 1 \tag{27}$$

For example, if $\lambda_t = 1$ then $\eta_t = t$ and the unbiased covariance estimate is $\frac{1}{t} R_t^{\varepsilon}$. The unbiased covariance estimate R_t^{ε}/η_t obeys the following recursion:

$$\frac{R_t^{\varepsilon}}{\eta_t} = \frac{\lambda_t \ \eta_{t-1}}{\eta_t} \ \frac{R_{t-1}^{\varepsilon}}{\eta_{t-1}} + \frac{1}{\eta_t} \ \varepsilon_t \varepsilon_t^T =$$

$$= (1 - 1/\eta_t) \ \frac{R_{t-1}^{\varepsilon}}{\eta_{t-1}} + \frac{1}{\eta_t} \ \varepsilon_t \varepsilon_t^T \tag{28}$$

Let $w_t = 1/\eta_t$, and rename R_t^{ε}/η_t as R_t^{ε} to get Eq. (25a). Note also that Eq. (27) can be rewritten as (25b).

Instead of using Eq. (25) and factoring it at each time-step to get the square-root $R_t^{\varepsilon/2}$, it is possible to update directly $R_t^{\varepsilon/2}$. Note that

$$\underbrace{\begin{bmatrix} \sqrt{1-w_t} \ R_{t-1}^{\varepsilon/2} \\ \\ \sqrt{w_t} \ \ \varepsilon_t^T \end{bmatrix}}_{\text{pre-array}} \sim \underbrace{\begin{bmatrix} R_t^{\varepsilon/2} \\ \\ 0 \end{bmatrix}}_{\text{post-array}} \tag{29}$$

where ~ denotes equivalance up to an orthogonal transformation. Note that the right-hand-side matrix is upper triangular and can be obtained from the left-hand-side matrix by applying a Householder triangularization routine (see appendix D for details). The complete alorithm for computing $R_t^{\varepsilon/2}$ is as follows:

update w_t (25b)
compute $\sqrt{1-w_t}\, R_{t-1}^{\varepsilon/2}$, $\sqrt{w_t}\, \varepsilon_t$ and set them in a pre-array
apply a Householder triangularization routine (L=p+1, M=N=p)
the upper p x p matrix in the post-array is $R_t^{\varepsilon/2}$

To ensure the convergence of the RML algorithm it is necessary to monitor the stability of the polynomial det B(z) as explained in section 2.1. Target parameters can finally be obtained from the estimated transfer function matrix H(z) (24), as explained in Appendix B. In our simulations we estimated the spectrum of Target i by computing the power spectrum of the impulse response of H(z) from its i-th input. The TDOA's are computed by cross-correlating impulse responses at different outputs.

3.2 SIMULATION RESULTS

The multitarget tracking concept was tested by computer simulation. Some preliminary results are presented here. All of the tests so far were for fixed (non-moving) targets with stationary spectra. The RML algorithm described in section 3.1 was used to estimate the ARMA coefficients. At this stage, no attempt was made to enforce the special structure of the transfer function H(z). Therefore, uniqueness is guaranteed only for test cases corresponding to stable and minimum phase systems. As discussed in appendix B, the more general case requires the enforcement of the special structure of N(z) and D(z).

All of the following test cases were run on N=1024 data points and an input signal to noise ratio (at each receiver) of 20dB.

Case 1

Target No. 1

Spectral parameters: $d_1(z) = 1 - 1.64z^{-1} + 0.95z^{-2}$

Location parameters: TDOA = 2

Target No. 2

Spectral parameters: $d_2(z) = 1 - 1.34z^{-1} + 0.95z^{-2}$

Location parameters: TDOA = -2

$$H(z) = \begin{bmatrix} 1 & .9z^{-2} \\ .9z^{-2} & 1 \end{bmatrix} \begin{bmatrix} d_1(z) & 0 \\ 0 & d_2(z) \end{bmatrix}^{-1}$$

Fig. 4 depicts the cross-correlation and spectra of the impulse responses of the estimated transfer function $\hat{H}(z)$. Note tnat the correct delay estimates were obtained (the peaks of the correlation function). Note also the high degree of spectral separation that was obtained! The estimated spectrum of target No. 1 still has a component of the second target, but that component is attenuated by more than 30dB. The labeling of the targets (No. 1 and No. 2) is, of course, arbitrary but the algorithm correctly associated a spectrum with a location (TDOA).

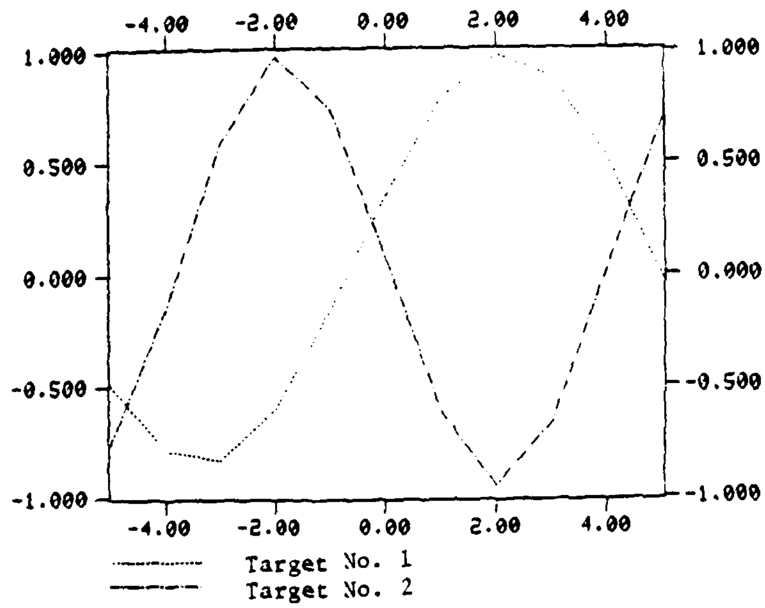


Figure 4a. Cross-correlation of the impulse responses corresponding to each target, N=1024 data points, k=1.0 .

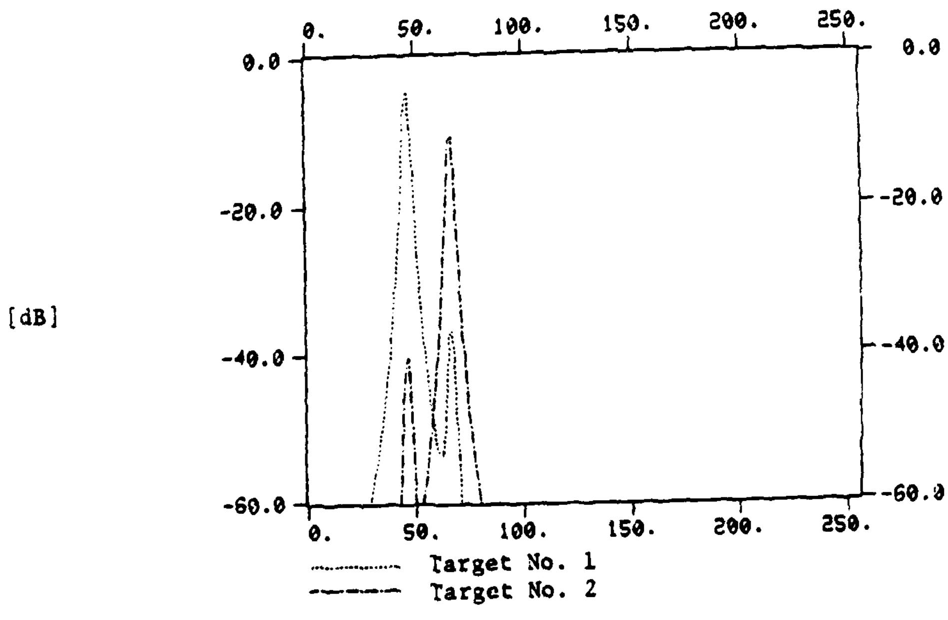


Figure 4b. The estimated spectra of two targets N=1024 data points, k=1.0 .

Case 2

Target No. 1

Spectral parameters: $d_1(z) = 1 - 1.64z^{-1} + 0.95z^{-2}$

Location parameters: TDOA = -2

Target No. 2

Spectral parameters: $d_2(z) = 1 - 0.95z^{-1} + 0.95z^{-2}$

Location parameters: TDOA = -2

$$H(z) = \begin{bmatrix} 1 & .9z^{-2} \\ .9z^{-2} & 1 \end{bmatrix} \begin{bmatrix} d_1(z) & 0 \\ 0 & d_2(z) \end{bmatrix}^{-1}$$

The results are depicted in Fig. 5.

Case 3

Target No. 1

Spectral parameters: $d_1(z) = 1 - 1.39z^{-1} + 0.8z^{-2}$

Location parameters: TDOA = 2

Target No. 2

Spectral parameters: $d_2(z) = 1 + 0.8z^{-2}$

Location parameters: TDOA = -2

$$H(z) = \begin{bmatrix} 1 & .9z^{-2} \\ .9z^{-2} & 1 \end{bmatrix} \begin{bmatrix} d_1(z) & 0 \\ 0 & d_2(z) \end{bmatrix}^{-1}$$

These targets had broader spectra then the targets in Case 1 and Case 2. The results are depicted in Fig. 6.

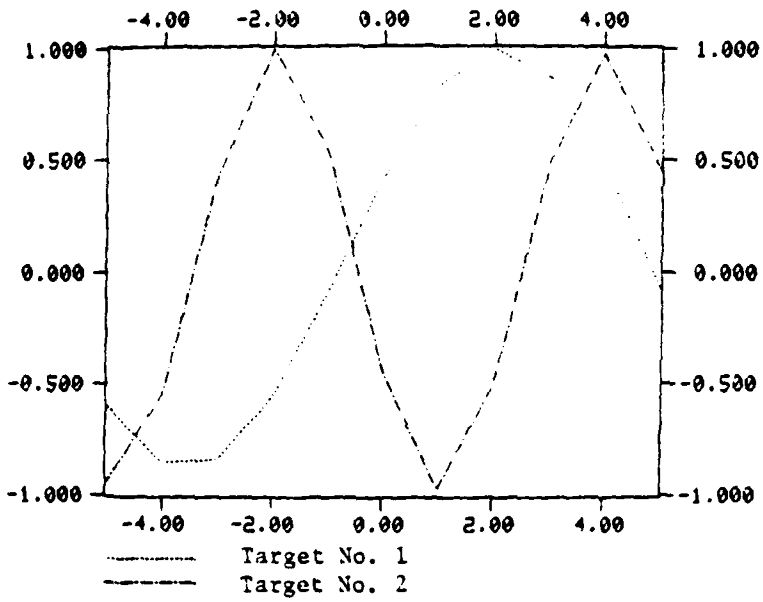


Figure 5a. Cross-correlation of the impulse responses corresponding to each target N=1024 data points, k=.9 .

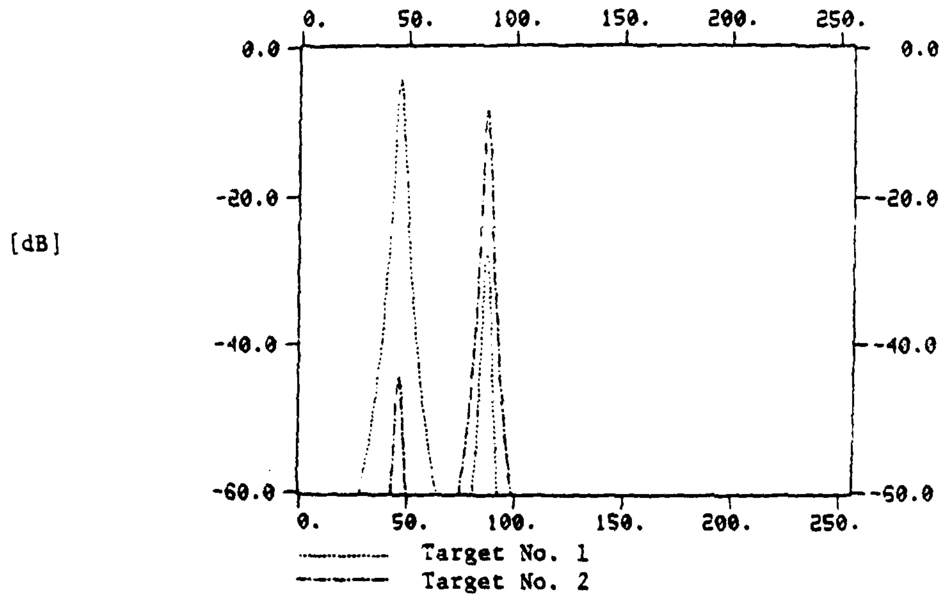


Figure 5b. The estimated spectra of two targets N=1024 data points, k=.9 .

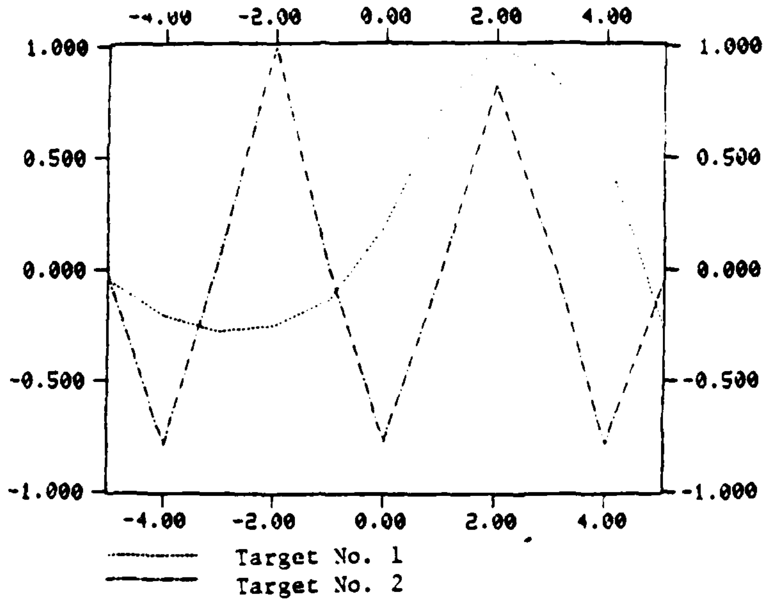


Figure 6a. Cross-correlation of the impulse responses corresponding to each target N=1024 data points, k=0 .

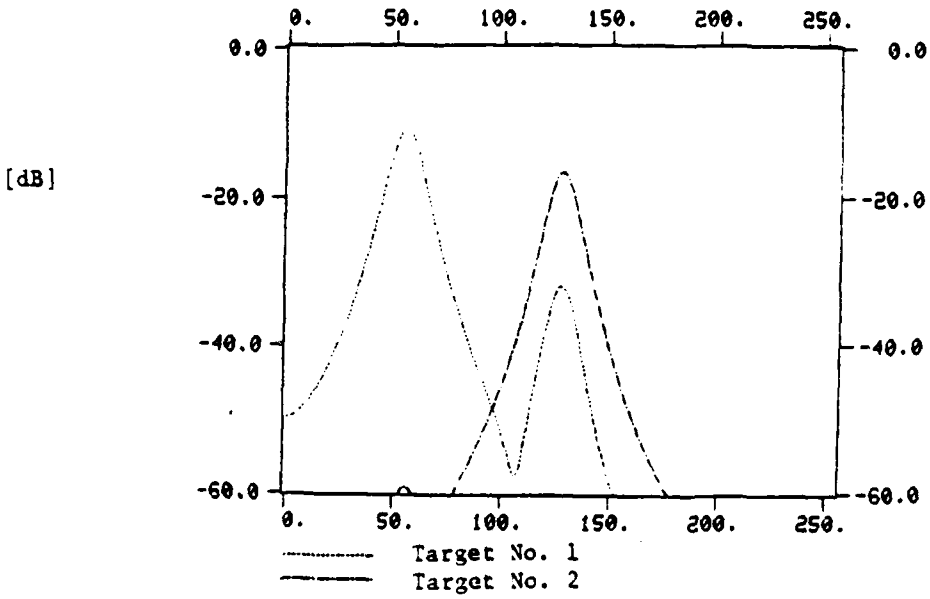


Figure 6b. The estimated spectra of two targets N=1024 data points, k=0 .

Case 4

Target No.1

Spectral parameters: $d_1(z) = 1 - 1.64z^{-1} + 0.95z^{-2}$

Location parameters: TDOA = 0

Target No.2

Spectral parameters: $d_2(z) = 1 - 1.34z^{-1} + 0.95z^{-2}$

Location parameters: TDOA = -1

$$H(z) = \begin{bmatrix} 1 & .9z^{-2} \\ 1 & .9z^{-1} \end{bmatrix} \begin{bmatrix} d_1(z) & 0 \\ 0 & d_2(z) \end{bmatrix}^{-1}$$

The results are depicted in Fig. 7.

In all the test cases so far we have chosen $N(z)$ so that $H(z)$ is a minimum phase plant. When $H(z)$ is non-minimum phase, the estimated transfer function $\hat{H}(z)$ is a transformed version of the true transfer function, i.e., $\hat{H}(z) = H(z)T(z)$, $T(z)$ = a para-unitary matrix. Thus, the spectral and location parameters can not be read directly from the impulse response. The following test illustrates this fact.

Case 5

Target No.1

Spectral parameters: $d_1(z) = 1 - 1.64z^{-1} + .95z^{-2}$

Location parameters: TDOA = 1

Target No. 2

Spectral parameters: $d_2(z) = 1 - 1.34z^{-1} + .95z^{-2}$

Location paramets: TDOA =2

$$H(z) = \begin{bmatrix} 1 & 1 \\ .9z^{-1} & .9z^{-2} \end{bmatrix} \begin{bmatrix} d_1(z) & 0 \\ 0 & d_2(z) \end{bmatrix}^{-1}$$

The results are depicted in Fig. 8. Note the loss of spectral seperation and the somewhat ambiguous delay estimates.

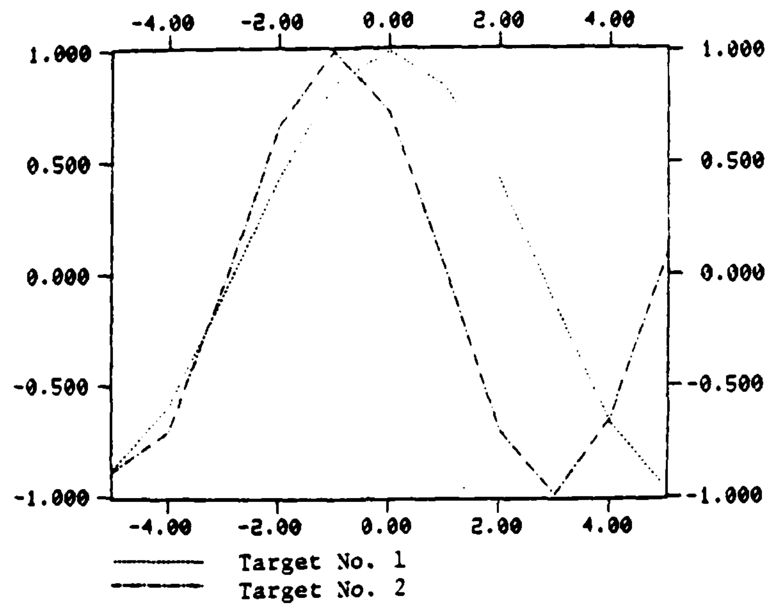


Figure 7a. Cross-correlation of the impulse responses corresponding to each target N=1024 data points, k=.9 .

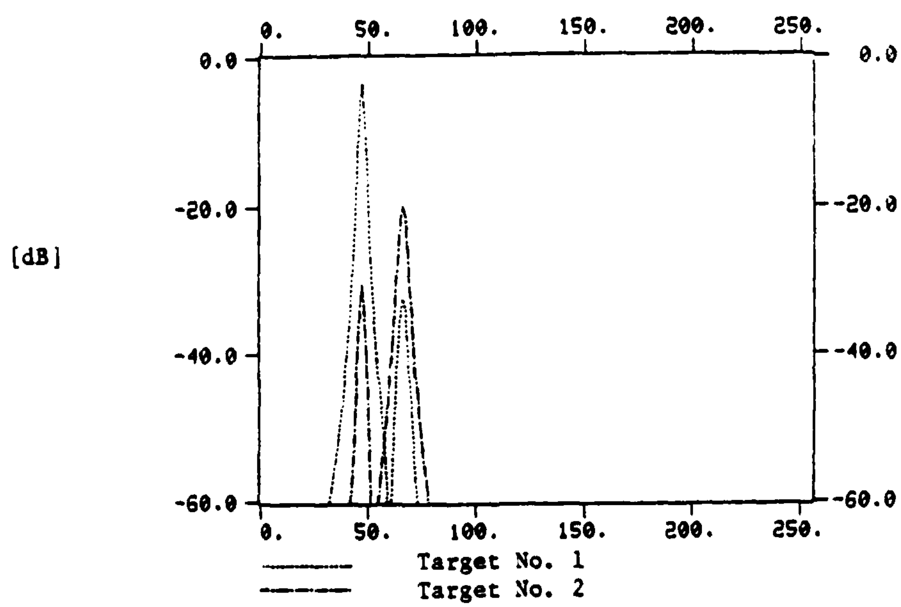


Figure 7b. The estimated spectra of two targets N=1024 data points, k=.9 .

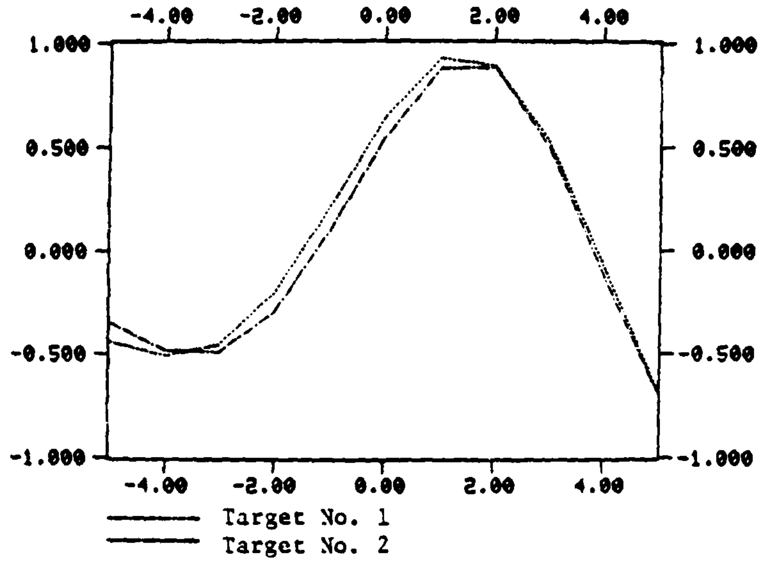


Figure 8a. Cross-correlation of the impulse responses corresponding to each target N=1024 data points, k=1.0 .

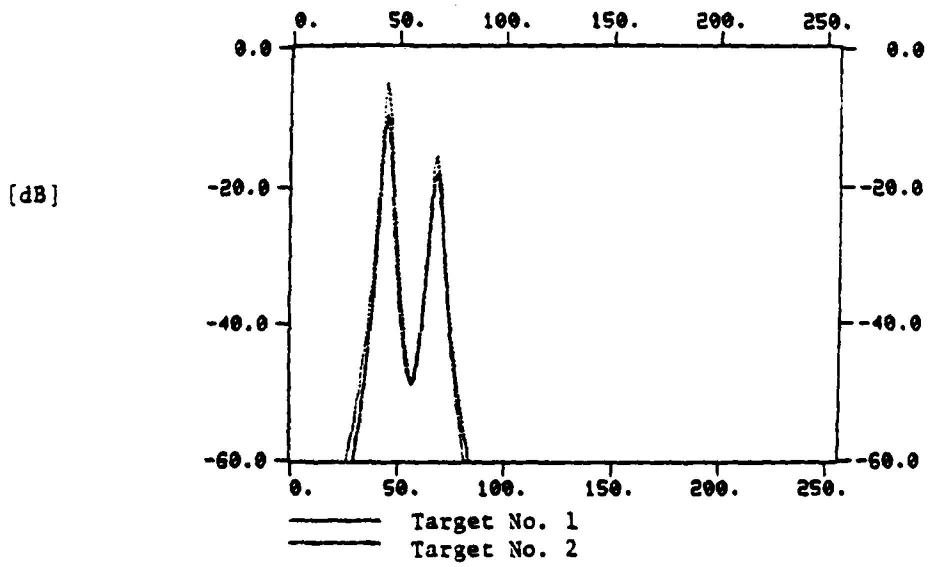


Figure 8b. The estimated spectra of two targets N=1024 data points, k=1.0 .

The results presented above are by no means conclusive. Only a limited number of test cases were run since the multichannel program is difficult to use in its present form. Also, further work is needed to solve adequately the uniqueness problem. The LMFD representation $\hat{A}_t^{-1}(z)\ \hat{B}_t(z)\ R_t^{-\varepsilon/2}$ has to be converted into an RMFD with the appropriate structure in order to get the correct TDOA's and spectral estimates in all situations (cf. test case no. 5). Better yet, an algorithm for estimating directly the RMFD needs to be derived.

4. WORK IN PROGRESS

In phase II we have completed the development of the single channel algorithm. While some issues such as convergence rate and tracking capability are still not completely resolved, the algorithm is basically ready for testing on real data and for a conclusive performance evaluation. This, however, is beyond the scope of the MTS project. The single channel algorithm will not be pursued further at this point.

The main objective of our current work and its continuation in phase III is the development and testing of the multitarget algorithm. While significant progress was made in phase II, the multitarget algorithm is not yet fully developed, as was discussed in Section 3. We are currently pursuing several lines of investigation:

(i) Analysis of the multitarget case.

The nonuniqueness problem and its physical interpretation are being studied. We are looking for ways of estimating correctly target spectra and TDOA's from the LMFD parameters. The problem of modeling Doppler effects and incorporating them in the MTS processor is being considered. Another issue is the determination of the number of targets and handling non-square transfer function matrices.

(ii) Refinement of the multichannel RML algorithm.

The algorithm described in Section 3.1 has been tested under high SNR conditions. Its performance is severely degraded at low SNR's. Various methods for making the algorithm more robust will be explored. This will require the enforcement of some structural constraints to reduce the number of parameters involved.

(iii) Alternative methods.

The "ideal" approach to the multitarget problem would be to estimate directly the parameters of a physical model (i.e. the RMFD, for the situation assumed in this report, or a more general model in case doppler effects are

included). A possible approach which circumvents some of the difficulties involved in estimating the RMFD parameters is to work with the spectral matrix $S(z) = H(z)\,H^T(z^{-1})$. Another possibility is to develop a suboptimal maximum likelihood type algorithm for estimating directly the RMFD parameters.

Due to limited resources we will not be able to investigate all of these issues. At the start of phase III the most promising direction will be chosen and most of our effort will be devoted to it. We plan to develop the multichannel algorithm sufficiently so that by the end of phase III it will be possible to draw conclusions regarding the feasibility of our parametric modeling approach.

5. PROJECT PUBLICATIONS

This section lists all the publications resulting from the Multitarget Tracking Studies project to date.

Conference Presentations and Publications

1. B. Friedlander and J.J. Anton, "System Identification for Multitarget Tracking", Proceeding of the 13th Asilomar Conference on Circuits Systems and Computers, Pacific Grove, California, November 1979. Also presented to the National Academy of Sciences Panel on Applied Mathematics Research Alternatives for the U.S. Navy, Washington D.C., November 2, 1979.

2. B. Friedlander, "An ARMA Modeling Approach to Multitarget Tracking," Proc. of the 19th IEEE Conference on Decision and Control, December 1980.

3. B. Friedlander, "A Pole-Zero Lattice Form for Adaptive Line Enhancement," Proc. of the 14th Asilomar Conference on Circuits, Systems and Computers, Pacific Grove, California, November 1980.

4. B. Friedlander, "System Identification Techniques for Adaptive Noise Cancelling," Proc. of the 14th Asilomar Conference on Circuits, Systems and Computers, Pacific Grove, California, November 1980.

5. B. Friedlander and M. Morf, "Least-Squares Algorithms for Adaptive Linear-Phase Filtering," Proc. of the 1981 IEEE Intl. Conf. on Acoustics, Speech and Signal Processing, Atlanta, Georgia, March 30 - April 1, 1981.

6. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Line Enhancement," Proc. of the 1981 IEEE Intl. Conf. on Acoustics, Speech and Signal Processing, Atlanta, Georgia, March 30 - April 1, 1981.

7. B. Friedlander, "A Modified Lattice Algorithm for Deconvolving Filtered Impulsive Processes," Proc. of the 1981 IEEE Intl. Conf. on Accoustics, Speech and Signal Processing, Atlanta, Georgia, March 30 - April 1, 1981.

8. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Spectral Estimation," Proc. Conference on Information Sciences and Systems, Johns Hopkins University, April, 1981.

9. B. Friedlander, "Application of Recursive Parameter Estimation Algoritms to Adaptive Signal Processing," Proc. Workshop on Adaptive Processing, Yale University, Connecticut, May, 1981.

10. B. Friedlander, "A Modified Pre-filter for Some Recursive Parameter Estimation Algorithms", Proc. 20th IEEE Conf. on Decision and Control, San Diego, California, December 16-18, 1981.

Submitted for Journal Publication

1. B. Friedlander, "Lattice Implementations of the Adaptive Line Enhancer," IEEE Trans. on ASSP.

2. B. Friedlander and M. Morf, "Least-Squares Algorithms for Adaptive Linear-Phase Filtering", IEEE Trans. on ASSP, to appear.

3. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Line Enhancement," IEEE Trans. on ASSP, to appear.

4. B. Friedlander, "System Identification Techniques for Adaptive Noise Cancelling", IEEE Trans. on ASSP.

5. B. Friedlander, "A Modified Lattice Algorithm for Deconvolving Filtered Impulsive Processes," IEEE Trans. on ASSP.

6. B. Friedlander, "A Modified Pre-Filter for Some Recursive Parameter Estimation Algorithms," IEEE Trans. on AC., to appear February 1982.

7. B. Friedlander, "System Identification Techniques for Adaptive Signal Processing," IEEE Trans. on ASSP., to appear. Will also be published in the Journal of Circuits, Systems and Signal Processing.

8. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Spectral Estimation," IEEE Trans. Information Theory, to appear, July 1982.

Reports

B. Friedlander, "Multitarget Tracking Studies: Phase I Final Report," Report No. 5334-01, July 1980.

REFERENCES

1. B. Friedlander, "Multitarget Tracking Studies: Phase I Final Report," Report No. 5334-01, July 1980.

2. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Line Enhancement," Proc. of the 1981 IEEE Intl. Conf. on Acoustics, Speech and Signal Processing, Atlanta, Georgia, March 30-April, 1981.

3. B. Friedlander, "System Identification Techniques for Adaptive Noise Cancelling", Proc. of the 14th Asilomar Conference on Circuits, Systems and Computers, Pacific Grove, California, November 1980.

4. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Spectral Estimation", IEEE Trans. Information Theory, to appear, July 1982.

5. L. Ljung, "Analysis of Recursive Stochastic Algorithms," IEEE Trans. on Automatic Control, Vol. AC-22, pp. 551-575, 1977.

6. B. Friedlander, "A Modified Pre-Filter for Some Recursive Parameter Estimation Algorithms", IEEE Trans. on AC, to appear, Febuary 1982.

7. C.L. Lawson and R.J. Hanson, Solving Least-Squares Problems, Prentice-Hall, Englewood Cliffs, New Jersey, 1976.

8. G.J. Bierman, Factorization Methods for Discrete Sequential Estimation, New York: Academic Press, 1977.

9. J.R. Treichler, "The Spectral Line Enhancer - The Concept, An Implementation and An Application", Ph.D. dissertation, Stanford University, June 1977.

10. B. Friedlander, "A Pole-Zero Lattice Form for Adaptive Line Enhancement", Proc. of the 14th Asilomar Conference on Circuits, Systems and Computers, Pacific Grove, California, November 1980.

11. L. Ljung, et. al. Recursive Identification, preprint 1981.

12. L. Marple, "A New Autoregressive Spectrum Analysis Algorithm", IEEE Trans. Acoustics Speech and Signal Processing, Vol. ASSP-28, No. 4, pp. 441-453, August 1980.

13. B. Friedlander and M. Morf, "Least-Squares Algorithms for Adaptive Linear-Phase Filtering," Proc. of the 1981 IEEE Intl. Conf. on Acoustics, Speech and Signal Processing, Atlanta, Georgia, March 30 - April 1, 1981.

14. B. Friedlander, "A Modified Lattice Algorithm for Deconvolving Filtered Impulsive Processes," Proc. of the 1981 IEEE Intl. Conf. on Acoustics, Speech and Signal Processing, Atlanta, Georgia, March 30 - April 1, 1981.

15. IEEE Trans. Acoustics Speech and Signal Processing, Special Issue on Time Delay Estimation, June 1981.

16. Y.T. Chan, J.M. Riley and J.B. Plant, "A Parameter Estimation Approach to Time-Delay Estimation and Signal Detection",IEEE Trans. Acoustics Speech and Signal Processing, Vol. ASSP-28, No. 1, pp. 8-16, February 1980.

17. P.E. Caines and L. Ljung, "Asymptotic Normality and Accuracy of Prediction Error Methods," Res. Rep. No. 7602, University . Toronto, Dept. Electrical Engineering, 1976.

18. K.J. Åström, "On the Achievable Accuracy in Identification Problems," Proc. of the 1967 IFAC Conference of Identification.

19. L. Ljung, "On Positive Real Functions and the Convergence of Some Recursive Schemes," IEEE Trans. on Automatic Control, Vol. AC-22, pp. 539-551, 1977.

20. K.J. Åström and T. Söderström, "Uniqueness of the Maximum Likelihood Estimates of the Parameters of an ARMA Model," IEEE Trans. Automatic Control, Vol. AC-19, pp. 769-773, December 1974.

21. T.M. Sullivan, P.L. Frost and J.R. Treichler, "High Resolution Spectral Estimation", Tech. Report, ARGO Systems, Inc., June 1978.

22. J.D. Markel and A.H. Gray Jr., Linear Prediction of Speech, Springer-Verlag, 1976.

23. B. Friedlander, "Recursive Algorithms for All-Pole Ladder Forms", Technical Report TM5334-02, Systems Control Inc., June 1980.

24. YOHO Advanced 2400 BPS Speech Compression, Progress Report II, Systems Control Inc., June 1980.

25. D. Morgan, "Equations of Programs for Pole-Zero Ladder Forms", TM5352-03, Systems Control Inc., May 1980.

26. H. Lev Ari, "Ladder Programs", Systems Control Inc., 1981.

APPENDIX A

SYSTEM IDENTIFICATION TECHNIQUES
FOR
ADAPTIVE SIGNAL PROCESSING

To appear in IEEE Trans. on Acoustics, Speech and Signal Processing. An expanded version of this paper will appear in the Journal of Circuits Systems and Signal Processing.

SYSTEM IDENTIFICATION TECHNIQUES FOR ADAPTIVE SIGNAL PROCESSING*

B. Friedlander

Systems Control, Inc.
1801 Page Mill Road
Palo Alto, CA 94304

ABSTRACT

Many problems in adaptive filtering can be approached from the point of view of system identification. The recursive maximum likelihood algorithm is proposed for estimating the parameters of the signal model. The parameter estimates are then used to form an adaptive infinite impulse response filter. Several examples are discussed including: adaptive line enhancement, adaptive deconvolution, adaptive noise cancelling and adaptive time delay esitmation.

*This work was supported in part by the Office of Naval Research, Contract No. N00014-79-C-0743.

1. INTRODUCTION

Considerable progress was made in the last decade in the development and analysis of recursive parameter estimation algorithms. The major part of this work was in the area of system identification, in the context of controlling plants with unknown or slowly time varying parameters. A large number of algorithms were developed for fitting linear models to the observed data. The following ARMAX model is an example of the class of models that are typically considered.

Let u_t and y_t denote the input and output processes of the model, and v_t an unmeasurable white noise process (i.e. a "disturbance", in the control terminology). These processes are related by the following equation:

$$y_t = -\sum_{i=1}^{NA} a_i y_{t-i} + \sum_{i=1}^{NB} b_i u_{t-i} + \sum_{i=1}^{NC} c_i v_{t-i} + v_t \qquad (1)$$

which can be written in polynomial form as

$$A(z^{-1}) y_t = B(z^{-1}) u_t + C(z^{-1}) v_t \qquad (2a)$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{NA} z^{-NA} \qquad (2b)$$

$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{NB} z^{-NB} \qquad (2c)$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \cdots + c_{NC} z^{-NC} \qquad (2d)$$

$$z^{-1} = \text{unit delay operator, i.e., } z^{-1} x_t = x_{t-1} \qquad (2e)$$

The case where $B(z^{-1}) = 0$ is of special interest in signal processing applications, since the output $y_t = (C(z^{-1})/A(z^{-1}))v_t$ is then an autoregressive moving-average (ARMA) process. When $B(z^{-1}) = 0$ and $C(z^{-1}) = 0$ we have an autoregressive (AR) process $y_t = (1/A(z^{-1}))v_t$. Such processes are very common in time series analysis and statistical signal processing.

Many problems in signal processing involve signals that can be represented by linear models of this type (AR, ARMA, ARMAX), as we will show later. Knowledge of the signal model parameters makes it relatively straightforward to design filters that perform various processing functions such as: linear prediction/smoothing, deconvolution, noise/interference suppression, or spectral analysis. When the signal model is not known, the parameters of the related filter need to be adaptively adjusted. A technique that is commonly used in adaptive control problems, is to first estimate the model parameters and then design the controller as if these estimates were the true parameter values.

The same idea can be used for adaptive signal processing, as already indicated in [1]-[4]. The combination of a recursive parameter estimation algorithm with a filtering technique based on known model parameters is the main theme of this paper.

The successful application of system identification techniques in adaptive control motivated us to apply the same techniques to signal processing. Of particular interest is the fact that some commonly used parameter estimation algorithms such as recursive maximum likelihood (RML), extended least-squares (ELS) or recursive generalized least squares (RGLS) [5]-[6], are capable of estimating ARMA (or ARMAX) parameters, and not just AR parameters. As we shall see, this leads naturally to adaptive infinite impulse response (IIR) filters. Adaptive IIR filtering is considered by the signal processing community to be a difficult problem. Consequently, the overwhelming majority of the work in the area of adaptive signal processing seems to concentrate on finite impulse response (FIR) filtering. The application of system identification algorithms opens the way to the development of a whole new class of adaptive IIR filters, backed by the extensive convergence analysis that was performed in recent years [7]-[12].

In spite of the natural interconnections between system identification and adaptive signal processing very little work seems to have been done to transfer algorithms from one area to the other. The objective of this paper is to report on some recent work in which a version of the RML algorithm was successfully applied to a number of problems including: adaptive line enhancement, adaptive deconvolution, adaptive noise cancelling and time-delay estimation. A brief description of these applications is presented in Section 3.

In Section 2 we present the RML algorithm and discuss its properties. Some of the special characteristics of the signal processing problem (when compared to the control problem) and their effect on the behavior of the algorithm are also discussed. Finally, in Section 4 we outline some alternative algorithms for adaptive processing and areas for further investigation. We hope that our work will stimulate further research into the numerous potential applications of parameter estimation algorithms to adaptive signal processing.

2. THE RECURSIVE MAXIMUM LIKELIHOOD ALGORITHM

The RML algorithm provides a recursive estimate of the parameters of the ARMAX model in equation (1). For a detailed derivation of this algorithm see [13] [14]. Here we present only a summary of the recursions.

Let θ denote the vector of model parameters

$$\theta = [a_1,\ldots,a_{NA},\ b_1,\ldots,b_{NC},\ c_1,\ldots,c_{NC}]^T, \tag{3a}$$

and ϕ_t, ψ_t denote the data vector and the filter data vector, respectively.

$$\phi_t = [-y_{t-1},\ldots,-y_{t-NA}, u_{t-1},\ldots,u_{t-NB}, e_{t-1},\ldots,\bar{e}_{t-NC}]^T \tag{3b}$$

$$\psi_t = [-\bar{y}_{t-1},\ldots,-\bar{y}_{t-NA}, \bar{u}_{t-1},\ldots,\bar{u}_{t-NB}, \bar{e}_{t-1},\ldots,\bar{e}_{t-NC}]^T \tag{3c}$$

Denote by $\hat{\theta}_t$ the parameter estimates at time t, and by $\hat{C}(z^{-1})$ the filter whose coefficents are the estimates $\hat{c}_i$. Then,

$$\varepsilon_{t+1} = y_{t+1} - \phi^T_{t+1}\hat{\theta}_t = \text{prediction error} \tag{4a}$$

$$P_{t+1} = [P_t - P_t\psi_{t+1}\psi^T_{t+1}P_t/(\lambda_{t+1} + \psi^T_{t+1}P_t\psi_{t+1})]/\lambda_{t+1} =$$

$$= \text{error covariance matrix} \tag{4b}$$

$$\hat{\theta}_{t+1} = \hat{\theta}_t + P_{t+1}\psi_{t+1}\varepsilon_{t+1} \tag{4c}$$

$$e_{t+1} = y_{t+1} - \phi^T_{t+1}\hat{\theta}_{t+1} = \text{residual} \tag{4d}$$

$$\left.\begin{aligned} \bar{y}_t &= (1/\hat{C}(kz^{-1}))y_t \\ \bar{u}_t &= (1/\hat{C}(kz^{-1}))u_t \\ \bar{e}_t &= (1/\hat{C}(kz^{-1}))e_t \end{aligned}\right\} \text{filtered quantities} \tag{4e}$$

with initial conditions

$P_0 = \alpha I$, α = initial estimate of the covariance

$\hat{\theta}_0 = \theta_0$, initial estimate of θ (typically $\theta_0 = 0$).

The parameter λ_t is an exponential weighting on the data. Typically λ_t is a constant close to unity, or

$$\lambda_{t+1} = \lambda_0\lambda_t + (1-\lambda_0)\ (\lambda_0 = .99,\ \lambda_0 = 0.95) \tag{5}$$

The parameter k is used to "pull in" the roots of the polynomial $\hat{C}(kz^{-1})$ into the unit circle, when $\hat{C}(z^{-1})$ has roots near the unit circle, as is often the case in signal processing applications. This parameter affects the convergence rate of the algorithm as discussed in [15]. In fact, for k=0 this algorithm becomes the so-called Extended Least-Square algorithm described in [16], [17]. In most cases the choice of k is not critical and in the following we assume that k is close or equal to unity. To ensure convergence, the stability of $\hat{C}(z^{-1})$ needs to be monitored. If unstable, the parameter estimates need to be projected into a region of stability [7],[8].

The asymptotic properties of the RML algorithm have been investigated in considerable detail. It was shown that asymptotically the recursive maximum likelihood technique has the same properties as the corresponding off-line version. Thus nothing is sacrificed by going to a recursive implementation, provided that enough data is available. The maximum likelihood estimator has all the desirable properties one may expect from a parameter estimator:

- Asymptotic consistency [18], i.e., $\hat{\theta}_N \to \theta$ as the number of data points N goes to infinity
- Asymptotic efficiency [19], i.e., the estimation error covariance approaches the Cramer-Rao lower bound
- The estimation error distribution is asymptotically normal [19].

The convergence properties of the RML algorithm were studied by Ljung [7],[8] and others [5],[12]. It was shown that this algorithm will always converge to a local maximum of the likelihood function. In some situations there may exist "false" maxima which can cause difficulties. However, for ARMA processes it was shown that all the local maxima coincide with the global maximum [20] (provided that the orders (NA,NC) of the estimated ARMA model are equal to or larger than the true model orders).

Relatively little is known about the convergence rate of the RML algorithm for different types of processes. Hardly any analytical units are available and most of the results are based on extensive simulation studies [11]. However, most of these studies were

related to control problems involving stable plants with poles and zeroes well inside the unit circle. Models arising in signal processing applications typically involve narrowband signals, which are represented by poles and zeroes on or very near the unit circle. Our own experience indicates that pole and zero locations have a significant effect on convergence rates. When the poles and zeroes are well separated, fast convergence was observed. Clusters of poles and zeroes, especially when they are near the unit circle, often lead to much slower convergence. The issue of convergence rate is crucial in adaptive signal processing since signals are often nonstationary and time-varying and it is important to know how well the adaptive filter will track the changing parameters. In adaptive control problems the parameters are often very slowly time varying (compared to the time constants of the plant). We are currently investigating the convergence rate of the RML algorithm for ARMA models with poles and zeroes near the unit circle.

The properties of the maximum likelihood estimator described above make it very attractive for adaptive IIR filtering, as illustrated in the next section.

3. ADAPTIVE SIGNAL PROCESSING: SOME EXAMPLES

3.1 The Adaptive Line Enhancer (ALE)

The ALE is an adaptive filter for narrowband signals in additive noise [21]-[23]. The ALE can be interpreted as an adaptive predictor, i.e., its output is $\hat{y}_{t|t-1}$: the estimate of y_t based on data up to time t-1. A narrowband (autoregressive) signal in white noise can be represented as an ARMA process [24]. Thus, the optimal predictor is given by

$$\hat{y}_{t|t-1} = -\sum_{i=1}^{NA} a_i y_{t-i} + \sum_{i=1}^{NC} c_i \varepsilon_{t-i} \quad (6)$$

which is depicted as a tapped delay line filter in Figure 1. The parameters of the filter will be adjusted using the RML algorithm (with NB=0). Note that the resulting filter has an infinite impulse response while most ALE-s discussed in the literature are of the FIR type. In [24] we discuss in detail the advantages of the IIR-ALE and its superior performance at low signal-to-noise ratios.

To illustrate the behavior of the IIR-ALE we depict in Figure 2 the input and output of the filter for a single sinusoid in noise, at SNR = 0 dB. The spectra were obtained by a 512 point FFT. The ALE

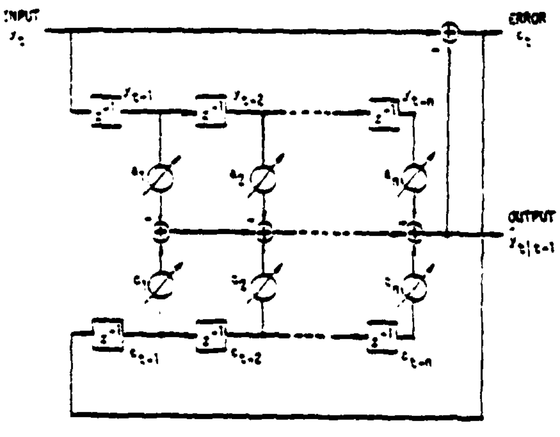


Figure 1 The IIR Adaptive Line Enhancer

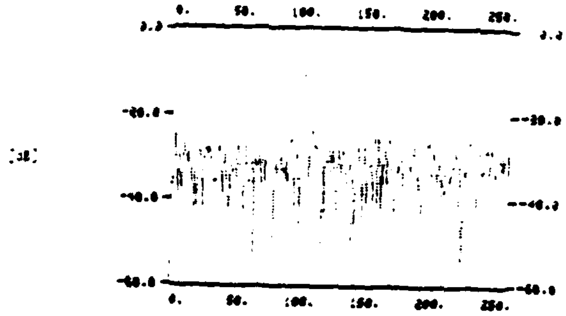


Figure 2a Spectrum of the ALE Input y_t

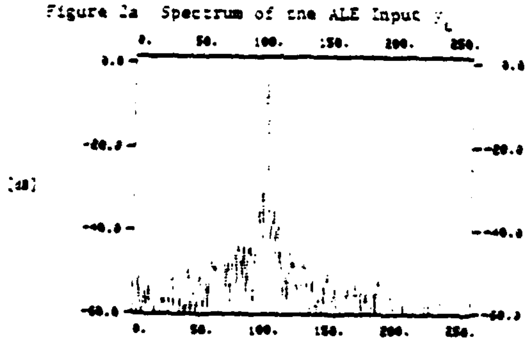


Figure 2b Spectrum of the ALE Output $\hat{y}_{t|t-1}$

output data corresponds to the last 512 samples in a total of 2048 data points. Note the significant noise reduction. Further noise suppression is achieved if the filter is allowed to continue its convergence. See [24],[25] for more results.

The parameters of the ARMA model $C(z^{-1})/A(z^{-1})$ which serve as the coefficients of the IIR-ALE can also be used to estimate the spectrum of the observed signal. In [26] we present some comparisons of the ARMA spectra obtained by the RML with corresponding estimates obtained by the maximum entropy method.

3.2 Adaptive Deconvolution

The need to extract a signal, given a filtered version of the signal arises in many situations including: (i) speech analysis/synthesis by linear predictive coding, and pitch estimation, (ii) estimation of the reflectivity sequence in seismic data processing, (iii) channel equalization for the removal of the intersymbol interference caused by convolution of the message sequence with the channel impulse response.

Consider the case where a white signal process passes through an IIR filter $C(z^{-1})/A(z^{-1})$ (stable and minimum phase). The RML algorithm can be used to estimate the filter parameters, and deconvolution will be achieved by passing the data through the estimated inverse filter $\hat{A}(z^{-1})/\hat{C}(z^{-1})$. Note that most current deconvolution techniques are limited to the case where the convolving filter has only poles ($1/A(z^{-1})$) while the RML can handle the pole-zero case ($C(z^{-1})/A(z^{-1})$). In Figure 3 we present a comparison between IIR and FIR deconvolution. The data y_t was generated in this case by passing a train of impulses through the filter

$$\frac{C(z^{-1})}{A(z^{-1})} = \frac{1 + z^{-2}}{1 + .606z^{-1} + .93z^{-2}} \quad (7)$$

and adding measurement noise. The signal to noise ratio was 20 dB. Note that the IIR filter restores the

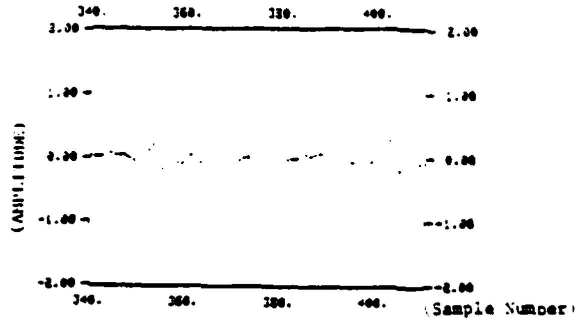


Figure 3a The Deconvolved Signal $\hat{s}_t$ at the Output of an IIR Filter (NA=2, NC=3)

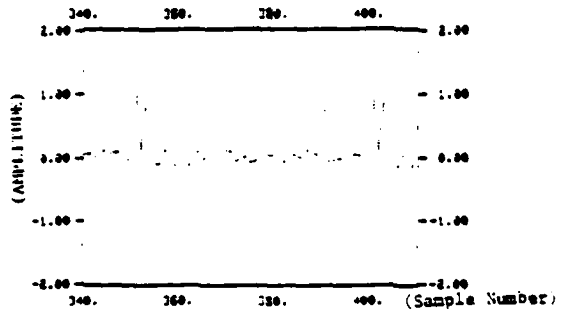


Figure 3b The Deconvolved Signal $\hat{s}_t$ at the Output of an FIR Filter (NA=2,NC=0)

original impulse sequence while the FIR filter gives pairs of impulses. This is caused by the fact that the moving average part of the convolving filter is not identified by the algorithm and therefore the deconvolved signal is $A(z^{-1})y_t = C(z^{-1})v_t = (1+z^{-2})v_t$. For a more detailed discussion of adaptive deconvolution see [27], [28].

3.3 Adaptive Noise Cancelling (ANC)

The ANC and its applications are discussed in [29] [30]. Here we present only a very brief description. In the ANC problem we are provided with a noisy measurement y_t of the signal s_t

$$y_t = s_t + z_t \tag{8}$$

and also with a reference input u_t which contains information about the noise process z_t. From the side information (u_t) an estimate $\hat{z}_t$ of the noise process is obtained and then subtracted from the primary input y_t to "cancel out" the noise, i.e.,

$$s_t = y_t - \hat{z}_t \tag{9}$$

under the assumptions that u_t and z_t are related by a linear model and that s_t is an ARMA process it can be shown [31] that y_t is an ARMAX process of the form

$$y_t = z_t + s_t = \frac{B(z^{-1})}{A(z^{-1})} u_t + \frac{C(z^{-1})}{A(z^{-1})} v_t \tag{10}$$

where u_t is the reference input and v_t is a white noise process. The noise estimate can be obtained by the following IIR filter.

$$\hat{z}_t = -\sum_{i=1}^{NA} a_i \hat{z}_{t-i} + \sum_{i=1}^{NB} b_i u_{t-i} \tag{11}$$

Using the RML algorithm to estimate the ARMAX model parameters and then estimating the "clean" signal by equations (9), (11) gives an IIR-ANC. Figure 4 depicts the noise cancellation achieved for a narrowband signal in the presence of narrowband interference. See [31] for more results.

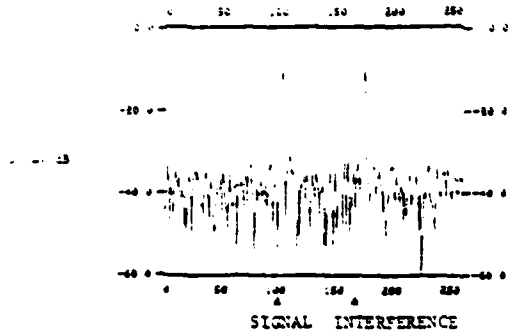


Figure 4a Spectrum of Primary Input y_t, Narrowband Interference

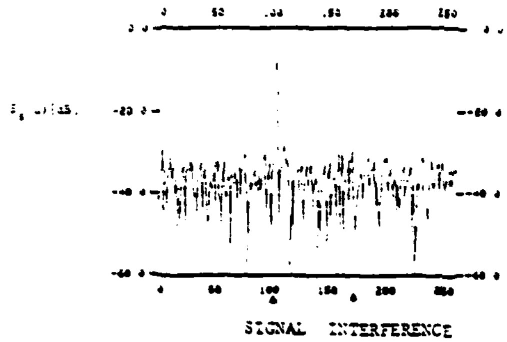


Figure 4b Spectrum of ANC Output s_t

Note that in addition to obtaining the predictor $B(z^{-1})/A(z^{-1})$ of the noise process z_t, the RML algorithm automatically provides us with an ARMA model $C(z^{-1})/A(z^{-1})$ for the signal process s_t. If the signal s_t is a narrowband process corrupted by white noise (uncorrelated with the noise process z_t measured by the reference input), additional noise suppression can be obtained by narrowband filtering. Thus, the ARMA parameters (C,A) can be used to form an IIR-ALE as was discussed in Section 3.1, i.e.,

$$\hat{s}_t = -\sum_{i=1}^{NA} a_i s_t + \sum_{i=1}^{NC} c_i \varepsilon_{t-i} \tag{12}$$

In otherwords, the RML algorithm leads naturally to an adaptive filter (equations (9), (11), (12)) that can be interpreted as an IIR-ANC followed by an IIR-ALE. In [31] we present a much more detailed discussion of this filter.

3.4 Adaptive Time Delay Estimation

The need to estimate time delay between two signals arises in many applications such as target localization by sonar systems, and position estimation by radio navigation systems. The problem is usually formulated as follows: let us assume that two sensors

(recivers, microphones, geophones) receive time shifted and scaled versions of some signal x_t: $u_t = x_t + n_t$, $y_t = dx_{t-\tau} + m_t$, where m_t, n_t are independent measurement noise processes. Many different techniques for estimating the delay τ have been proposed in the literature. These techniques typically involve filtering the signals and then cross-correlating [32]-[34]. To do this in an optimal way (in the least-squares or maximum likelihood sense) requires knowledge of the statistics of both the signal and noise. Using the system identification approach we were able to derive an adaptive technique for time delay estimation which requires no prior information [27],[35]. Here we present only the simplest version of the algorithms presented in [27],[35].

Note that two processes that are delayed versions of the same underlying signal are related by a moving-average filter whose coefficients contain the delay information. In the example mentioned above

$$y_t = B(z^{-1})u_t + v_t \qquad (13)$$

where

$$B(z^{-1}) = dz^{-\tau}$$

$$(b_i=0 \text{ for } i\neq\tau \text{ and } b_i=d \text{ for } i=\tau)$$

$$v_t = m_t - dn_{t-\tau} = \text{a white noise process}$$

Using the RML algorithm to estimate the coefficients of $B(z^{-1})$ will provide an estimate of the time delay by looking at the index of the largest coefficient of $B(z^{-1})$. If the delay is not an integer multiple of the sampling interval, it is necessary to perform a simple interpolation to get at the true delay [27]. The RML algorithm is, of course, capable of handling the case where v_t is not white.

A much more sophisticated approach is based on the idea of fitting a <u>multichannel</u> model to a vector of sensor measurements y_t, e.g.,

$$y_t = -\sum_{i=1}^{NA} A_i y_{t-i} + \sum_{i=1}^{NC} C_i v_{t-i} \qquad (14)$$

where y_t is a $p \times 1$ vector and A_i, C_i are $p \times p$ matrices. In [27], [36] we have shown that the resulting multi-input multi-output (MIMO) adaptive filter can be interpreted as a combination of an adaptive beamformer and a time delay estimator. The interesting point brought out in [36] is that this filter is capable of handling simultaneously several (up to p) targets! Here we only note that the use of MIMO signal models opens the way for developing new classes of MIMO adaptive filters which have numerous applications in array processing, beamforming, and processing of multisensor data. Current adaptive filtering techniques are almost entirely devoted to single input single output filters. We believe that perhaps the most important contribution of our modeling approach is that it provides a systematic framework for handling multichannel problems.

4. CONCLUSIONS

We presented an approach for developing adaptive filters for various signal processing problems. While the RML algorithm described in this paper is well known, its application to the class of problems described in Section 3 is apparently new. The RML algorithm was presented here in one particular form. It is important to note that alternative forms can be used to derive other implementations of these adaptive filters with similar asymptotic properties. Some examples:

<u>Square-Root Form</u>

The update equation (4) for the error covariance matrix P_t suffers from numerical problems when the number of estimated parameters $n = NA + NB + NC$ is large (e.g., $n > 10$). A much better implementation is obtained by using the square root form in which $P_t^{1/2}$ is propagated rather than P_t (where $P_t^{1/2} P_t^{T/2} = P_t$). The advantages of square-root algorithms were discussed in deail by Bierman [37].

<u>"Fast" Implementation</u>

The RML algorithm presented in equation (4) requires in the order of $4n^2 + 5n$ multiplications and additions per data point. For higher order models (i.e., large values of n) the amount of computation may become excessively large. Thus, it is important to search for more efficient estimation techniques. Using the idea of "shift low rank" developed by Morf led to implementations of the RML requiring proportional to n rather than n^2 multiplications and additions [38]. This technique provides an efficient way of computing the gain vector $P_t\varphi_{t+1}$. The rest of the algorithm remains unchanged. The detailed update equations can be found in [38] and will not be repeated here.

<u>Recursive Lattice Forms</u>

The recently developed square-root normalized lattice forms [39] combine the good numerical behavior of square-root implementation with computational efficiency. They furthermore provide a computational technique that is recursive both in time and in model order. In fact, the lattice form provides simultaneous parameter estimates corresponding to filters of all orders up to a maximum order! This is very sueful for addressing the order determination problem, which is one of the more difficult aspects of signal modeling.

<u>Symmetric $A(z^{-1})$ Polynomial</u>

In some situations the parameters of the ARMA model are interrelated in some way. For example, the case of sinusoids in white noise can be shown to have a symmetric $A(z^{-1})$ polynomial, i.e., $a_i = a_{n-i}$. In [40] we presented a way of incorporating this constraint in the parameter estimation algorithm. Several high resolution spectral estimation techniques are implicitly "symmetrizing" the predictor coefficients. In general, whenever the problem has some special structure that can be used to reduce the number of estimated parameters, one should explore the possibility of using that structure in the parameter estimation algorithm.

Finally we should note that the results presented here are only preliminary. Much work remains to be done on the analysis and performance evaluation of the RML and related algorithms in adaptive signal processing applications. Some specific issues that need to be addressed are: the convergence rate of the RML for different classes of signals, the tradeoff between parameter tracking capability (i.e., the value of λ) and filter performance, and the development of robust techniques for ARMA order determination in real-time.

<u>REFERENCES</u>

1. P. Hagander and B. Wittenmark, "A Self-tuning Filter for Fixed-Lag Smoothing," <u>IEEE Trans. on Inf. Theory</u>, Vol. IT-23, No. 3, pp. 377-384, May 1977.

2. B. Wittenmark, "A Self-Tuning Predictor," IEEE Trans. on Automatic Control, Vol. AC-19, No. 6, pp. 848-851, December 1976.

3. L. Ljung, "Convergence of an Adaptive Filter Algorithm," Int. J. Control, Vol. 27, No. 5, pp. 673-693, 1978.

4. J. Holst, "Adaptive Prediction and Recursive Estimation," Report LUTFD2/(TFRT-1013)/1-206/(1977), Dept. of Automatic Control, Lund University, Lund, Sweden, September 1977.

5. T. Söderström, L. Ljung and T. Gustavsson, "A Theoretical Analysis of Recursive Identification Methods," Automatica, Vol. 14, pp. 231-244.

6. G. C. Goodwin and R. L. Payne, Dynamic System Identification: Experiment Design and Data Analysis, Academic Press, 1977.

7. L. Ljung, "Analysis of Recursive Stochastic Algorithms," IEEE Trans. on Automatic Control, Vol. AC-22, pp. 551-575, 1977.

8. L. Ljung, "Convergence Analysis of Parametric Identification Methods," IEEE Trans. on Automatic Control, Vol. AC-23, pp. 770-783, 1978.

9. L. Ljung, "On Positive Real Functions and the Convergence of Some Recursive Schemes," IEEE Trans. on Automatic Control, Vol. AC-22, pp. 539-551, 1977.

10. L. Ljung, "Convergence of Recursive Estimators," Proceedings of the 5th IFAC Symposium on Identification and System Parameters Estimation, Darmstadt, 1979.

11. T. Söderström, L. Ljung and I. Gustavsson, "A Comparative Study of Recursive Identification Methods," Report 7427, Dept. Automatic Control, Lund, Sweden, 1974.

12. G. C. Goodwin and K. S. Sin, Adaptive Filtering Prediction and Control, Pre-print.

13. T. Söderström, "An On-Line Algorithm for Approximate Maximum Likelihood Identification of Linear Dynamic Systems," Report 7308, Dept. of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1973.

14. L. Ljung, et al., Recursive Identification, Pre-print, 1980.

15. B. Friedlander, "A Modified Pre-Filter for Some Recursive Parameter Estimation Algorithms," submitted to the IEEE-AC.

16. V. Panuska, "An Adaptive Recursive Least Squares Algorithm," Proc. IEEE Symposium on Adaptive Processes, The Conference on Decision and Control, 1969.

17. P. C. Young, "The Use of Linear Regression and Related Procedures for the Identification of Dynamic Processes," Proc. 7th IEEE Symposium on Adaptive Processes, UCLA, 1968.

18. L. Ljung, "On the Consistency of Prediction Error Methods," in System Identification, Advances and Case Studies, (R. K. Mehra and D. G. Lainiotis, eds.), Academic Press, New York, 1976.

19. P. E. Caines and L. Ljung, "Asymptotic Normality and Accuracy of Prediction Error Estimators," Res. Rep. No. 7602, Univ. of Toronto, Dept. of Electrical Engineering, 1976.

20. K. J. Åström and T. Söderström, "Uniqueness of the Maximum Likelihood Estimates of the Parameters of an ARMA Model," IEEE Trans. on Automatic Control, Vol. AC-19, pp. 769-773, December 1974.

21. L. J. Griffiths, "Rapid Measurement of Digital Instantaneous Frequency," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. ASSP-23, No. 2, pp. 207-222, April 1975.

22. J. R. Zeidler, E. H. Satorius, D. M. Chabries and H. T. Wexler, "Adaptive Enhancement of Multiple Sinoisoids in Uncorrelated Noise," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. ASSP-26, No. 3, pp. 240-254, June 1978.

23. J. R. Treichler, "The Spectral Line Enhancer - The Concept, An Implementation and An Application," Ph.D. disssertation, Stanford University, June 1977.

24. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Line Enahcement," Proc. of 1981 Intl. Conf. on Acoustics, Speech and Signal Processing, Atlanta, Georgia, March 30 - April 1, 1981.

25. B. Friedlander, "A Pole-Zero Lattice Form for Adaptive Line Enhancement," Proc. of the 14th Asilomar Conference on Circuits, Systems and Computers, Pacific Grove, California, November 1980.

26. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for ARMA Spectral Estimation," Proc. of the Conference on Information Sciences and Systems, March 25-27, 1981.

27. B. Friedlander, "Multitarget Tracking Studies: Phase I Final Report," Report No. 5334-01, Systems Control, Inc., July 1980.

28. B. Friedlander, "Recursive Parameter Estimation Algorithms for Adaptive Deconvolution," submitted for Publication.

29. B. Widrow, J. R. Glover, Jr., et al., "Adaptive Noise Cancelling: Principles and Applications," Proc. IEEE, Vol. 63, pp. 1692-1716, December 1975.

30. J. R. Glover, Jr., "Adaptive Noise Cancelling Applied to *Sinusoidal Interferences*," IEEE Trans. On Acoustics, Speech, and Signal Processing, Vol. ASSP-25, No. 6, December 1977.

31. B. Friedlander, "System Identification Techniques for Adaptive Noise Cancelling," Proc. of the 14th Asilomar Conference on Circuits, Systems and Computers, Pacific Grove, California, November 1980.

32. C.H. Knapp and G.C. Carter, "The Generalized Correlation Method for Estimation of Time Delay," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. ASSP-24, pp. 320-327, August 1976.

33. W. R. Hahn and S.A. Tretter, "Optimum Processing for Delay-Vector Estimation in Passive Signal Arrays," IEEE Trans. Inform. Theory, Vol. IT-19, pp. 608-614, September 1973.

34. J.C. Hassab and R.E. Boucher, "Optimum Estimation of Time-Delay by a generalized Correlator," IEEE Trans. on Acoustics, Speech, and Signal Processing, Vol. ASSP-27, No. 4, pp. 373-380, August 1979.

35. B. Friedlander, "A Recursive Maximum Likelihood Algorithm for Time Delay Estimation," submitted for publication.

36. B. Friedlander, "An ARMA Modeling Approach to Multitarget Tracking," Proc. of the 19th IEEE Conference on Decision and Control, December 1980.

37. G. J. Bierman, Factorization Methods for Discrete Sequential Estimation, New York: Academic Press, 1977.

38. L. Ljung, M. Morf and D. Falconer, "Fast Calculation of Gain Matrices for Recursive Estimation Schemes," Int. J. Control, Vol. 27, No. 1, pp. 1-19, 1978.

39. D.T. Lee, "Canonical Ladder Form Realizations and Fast Estimation Algorithms," Ph.D. dissertation, Stanford University, August 1980.

40. B. Friedlander and Morf, "Least Squares Algorithms for Adaptive Linear Phase Filtering," Proc. of 1981 ICASSP, Atlanta, Georgia, March 30 - April 1, 1981.

APPENDIX B

AN ARMA MODELING APPROACH
TO MULTITARGET TRACKING

An invited paper to the session on Data Correlation and Multiple Target Tracking, at the 19th IEEE Conference on Decision and Control, December 1980, Alberquerque, New Mexico. Published in the conference proceedings pp. 820-825.

AN ARMA MODELING APPROACH TO MULTITARGET TRACKING*

B. Friedlander
Systems Control, Inc.
1801 Page Mill Road
Palo Alto, California

ABSTRACT

A new approach is presented for locating multiple targets from signals received by a number of spatially distributed sensors. A multi-input multi-output model is assumed to generate the observed data. It is shown that the model parameters are related to the locations of the targets and to their spectra. The structure of the model is explored and a specific algorithm is given for estimating its parameters from sensor measurements. The main feature of the proposed approach is that it estimates simultaneously the parameters of all the targets of interest.

1. INTRODUCTION

Tracking multiple targets represents special difficulties since there can be uncertainties associated with the measurements beyond their inaccuracy which is usually modeled by some additive noise. This additional uncertainty is related to the origin of the measurements. Since several targets are present, it is necessary to sort out which measurement corresponds to which target. In other words, in addition to the problem of detection and bearing/range estimation, there is a problem of properly labeling the set of measurements. The latter problem is usually referred to as target association or track formation.

Typically, these two facets of multitarget tracking are treated separately. First, a set of potential target locations is obtained. Then some method is used to label these locations by the targets to which they correspond in a manner consistent with previous measurements. Techniques for labeling or multitarget tracking have been developed using various approaches including: Kalman filtering (for active sonar [1]; for radar [2]); Bayesian methods [3]-[7]; Integer programming [8]; and Track splitting [9], [10], [1], [2]. For a recent survey see [11].

In all of these techniques the basic detection and location estimation are performed separately for each target. The multitarget aspect of the problem enters only through the labeling procedure. In other words, the tracking problem is treated as a collection of single target problems, which has to be put together in a systematic and consistent way.

In this paper, we attempt to tackle directly the multichannel nature of the problem. Instead of looking at one target at a time, we want to estimate simultaneously multitarget parameters (such as location and spectrum). The approach is best understood in the context of the passive tracking, where an array of sensors measures signals (electromagnetic, acoustic or seismic) generated by targets. This type of problem arises in sonar systems, acoustic surveillance systems (detection of low flying aircraft) and seismic arrays (oil exploration, earthquake localization, intruder detection or artillary localization). The active tracking problem (radar, active sonar) can be handled in this framework but will not be discussed here.

The proposed approach is based on the idea of fitting a multi-input multi-output model to the vector time-series observed at the outputs of the sensors. Under certain assumptions the parameters of this model can be shown to contain information about the locations of all the targets, as well as other useful information such as target spectra. If the parameter estimation is performed recursively and continually, it is hoped that the relationship between the model parameters and the targets to which they correspond will be consistently maintained. Once the targets are labeled in a particular way, this labeling will stay fixed without need for rechecking or relabeling. This will eliminate the need for a separate step of target assoication which is inherent in current multitarget tracking techniques. In Section 2, we present the basic problem formulation and explore the relationship between model parameters and target location.

The formulation of target tracking as a multichannel signal modeling problem raises a number of interesting questions in the areas of system identification and structure of multivariable linear systems. Some of these issues are explored in Section 3, and some in Section 5. In Section 4, we present a specific algorithm for performing parameter estimation. This algorithm was used in a preliminary simulation study, which is described in the Appendix. This algorithm is presented here only as an example, and should not be interpreted as the best choice for this application.

Finally, we note that the emphasis in this paper is on developing a new framework for handling the multitarget tracking problem and exploring some of the theoretical issues it raises. No claims are made regarding the relative merits of this approach to current tracking algorithms. At this stage, we are mainly interested in treating directly the multichannel aspects of the tracking problem, and not in making a comparative evaluation with current techniques.

2. THE MODELING FRAMEWORK

To illustrate the basic ideas of our approach we consider first the single target case depicted in Figure 1. Two sensors are measuring the signal $x(t)$ propagating from a target located somewhere in the plane. We assume that the propagation involves only some time delays and attenuation. Thus, the outputs y_1, y_2 of the two sensors can be modeled as

$$y_1(t) = x(t-\tau_1) + v_1(t) \qquad (1a)$$

$$y_2(t) = \alpha x(t-\tau_2) + v_2(t) \quad , \qquad (1b)$$

where τ_1, τ_2 are the propagation delays from the target to the two sensors. α represents attenuation and v_1, v_2 are independent measurement noise processes. The time sampled version of these outputs will be written as:

$$y_1(k) = x(k-T_1) + v_1(k) \qquad (2a)$$

$$y_2(k) = \alpha x(k-T_2) + v_2(k) \quad , \qquad (2b)$$

where

$$t = k\Delta T, \quad \tau_1 = T_1\Delta T, \quad \tau_2 = T_2\Delta T.$$

Note that the delays τ_1, τ_2 are assumed to be integer multiples of the sampling period. The case of non-integer delays is discussed later.

*This work was supported in part by the Office of Naval Research, Code 431, Contract No. N00014-79-C-0743.

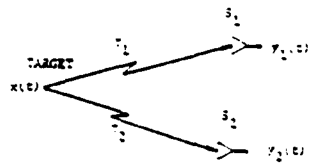


Figure 1. Two Sensors and One Target

In many applications, the singal $x(t)$ can be adequately modeled as an autoregressive (AR) process of the form

$$x(k) = -\sum_{i=1}^{p} d_i\, x(k-i) + u(k) \tag{3}$$

where $u(k)$ is a white noise process. This is particularly true for narrowband signals. The more general case of signals with rational (ARMA) spectra is discussed in Section 5. Taking the z-transform of Eq. (2) we get

$$y_1(z)=z^{-T_1}X(z)+V_1(z)=z^{-T_1}(1/d(z))U(z)+V_1(z) \tag{4a}$$

$$y_2(z)=nz^{-T_2}X(z)+V_2(z)=nz^{-T_2}(1/d(z))U(z) + V_2(z) \tag{4b}$$

where $d(z)=1+d_1z^{-1} + \ldots + d_pz^{-p}$. Written in vector form the transfer function from u to the sensor outputs y_1, y_2 is given by $N(z)/d(z)$, where

$$y(z)=N(z)X(z)+V(z)=N(z)(1/d(z))U(z)+V(z) \tag{6a}$$

and

$$N(z) = \begin{bmatrix} N_1(z) \\ N_2(z) \end{bmatrix} = \begin{bmatrix} z^{-T_1} \\ z^{-T_2} \end{bmatrix} \tag{6b}$$

Note that the numerator of this transfer function contains information about the relative delays from the target to the sensors (which is directly related to target location), while the denominator contains spectral information. Thus, if we could estimate the parameters of the model $N(z)/d(z)$ from the observed data $\{y_1(t), y_2(t)\}$, we could easily obtain the information needed for tracking. As long as we do not have direct measurements of $x(t)$ or $u(t)$ the only information that can be extracted from the spectral factorization problem $S_y(z)=(N(z)/d(z))(N(z^{-1})/d(z^{-1}))'$ is, of course nonunique. Arbitrary delays z^{-T} can be inserted into the spectral factor without changing the spectrum. Thus, as intuitively expected, the absolute delays T_1, T_2 cannot be determined. However, the <u>difference</u> in the degrees if the polynomials $N_1(z)$, $N_2(z)$ will be unique, and provide information about the time-differance of arrival (TDOA), $\Delta_{12}=T_1-T_2$. Given TDOA's for two or more pairs of sensors will uniquely locate the target [12].

The case where propagation delays (or their differences) are non-integer multiples of the sampling period can be approximated by an autoregressive moving-average (ARMA) model similar to equation (6). To see this consider the signal $x(t+\tau)$, and a sampling interval ΔT. Then,

$$x(t+\tau) = \sum_{k=-\infty}^{+\infty} x(k\Delta T)\ \mathrm{sinc}\ (t+\tau - k\Delta T) \tag{7a}$$

where

$$\mathrm{sinc}(t) = \sin\ (\pi t/\Delta T)/(\pi t/\Delta T)\ . \tag{7b}$$

Let $\tau = \ell\Delta T + \Delta\tau$, $0 \le \Delta\tau < \Delta T$, and let

$$y(t) = x(t+\tau) + v(t)\ . \tag{8}$$

Then the sampled version of $y(t)$ will be given by

$$y(i\Delta T) = \sum_{k=-\infty}^{\infty} x(k\Delta T)\ \mathrm{sinc}[(i+\ell-k)\Delta T+\Delta\tau]+v(k\Delta T) =$$

$$= \sum_{k=-\infty}^{\infty} x((k-i)\Delta T)\ \mathrm{sinc}[(\ell-k)\Delta T+\Delta\tau]+v(k\Delta T)$$

or using the notation $y(i\Delta T) \stackrel{\Delta}{=} y(i)$,

$$y(i) = \sum_{k=-\infty}^{\infty} n(k)\ x(i-k) + v(k) \tag{9}$$

where $n(k)=\mathrm{sinc}[(\ell-k)\Delta T+\Delta\tau]$. The coefficient $n(k)$ achieves its largest value for $k=\ell$ or $\ell\pm1$. As an approximation to (9) we may consider replaicng the infinite sum by a finite sum

$$y(i) = \sum_{k=0}^{m} n(k)\ x(i-k) + v(k) \tag{9'}$$

Thus, the polynomials $N_1(z)$ and $N_2(z)$ in (6) have to be replaced by

$$N_i(z) = \sum_{k=0}^{m} n_i(k)z^{-k},\quad i = 1, 2.$$

The delays T_1, T_2 can still be estimated by looking at the index of the largest coefficient, or by using interpolation. A reasonable estimate of the delay can be given by the value $\hat{\tau}$ which maximizes the function $n(\tau)$

$$n(\tau) = \sum_{k=0}^{m} n(k)\ \mathrm{sinc}\ (\tau-k) \tag{10}$$

The discussion above requires, of course, that the sampling rate $1/\Delta T$ be sufficiently large in relation to the signal bandwidth. The order of m of the moving average (MA) part of the model $N(z)$ will be determined by the sampling interval ΔT and by the expected spread of TDOA's. The latter depends on the maximum change in target bearing expected over the processing interval. It is possible to reduce this order by removing "bulk" delays prior to the parameter estimation algorithm. The order p of the AR part of the model $d(z)$ will depend on the number of spectral lines in the target spectrum (e.g., twice the number of important lines). In order to reduce the required sampling rate $1/\Delta T$ and the model order, it is assumed the signals have been shifted to a baseband (e.g., removal of carrier, if present). For a more detailed description of some of the practical issues involved see [13].

Note that the framework described here can also handle the effects of multipath propagation. In the presence of multipath $N_1(z)$ (and $N_2(z)$) will have several large coefficients corresponding to the direct delay and the multipath delays. Thus, a careful examination of the coefficients reveals useful information about the multipath structure. Since the direct path has the shortest delay, the <u>first</u> large coefficient of $N_i(z)$ will correspond to the direct path delay and provide TDOA information

The ARMA modeling approach can be extended to a multitarget environment. Here the system consisting of targets and sensors will be represented by a multi-output (MIMO) transfer function. A simple example is depicted in Figure 2. The equation describing the vector of measured data $y(k)$ is given by

$$y(z) = N(z)\ X(z) + V(z) \tag{11}$$

where $V(z)$ represents measurement noise and

$$N(z) = \begin{bmatrix} z^{-T_{11}} & z^{-T_{21}} \\ z^{-T_{12}} & z^{-T_{22}} \end{bmatrix} \quad \begin{matrix}\text{(or a more general} \\ \text{delay matrix)}\end{matrix} \tag{12}$$

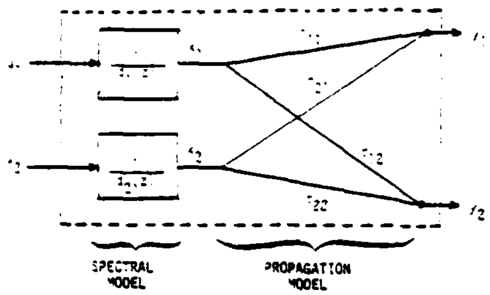


Figure 2. The Multitarget Case

The signal vector $X(z)$ is generated by

$$X(z) = D^{-1}(z)\,U(z), \quad D(z) = \begin{bmatrix} d_1(z) & 0 \\ 0 & d_2(z) \end{bmatrix}. \quad (13)$$

The output of the sensors in the multitarget case can be written as

$$Y(z) = N(z)\,D(z)^{-1}\,U(z) + V(z) \quad . \qquad (14)$$

Note that as in the single target case the numerator $N(z)$ contains location information (TDOA's) while the denominator $D(z)$ contains spectral information. If the parameters of $N(z)$, $D(z)$ are given the multi-target tracking problem is basically solved. The question is how to estimate these parameters from the data $y(t)$ in a unique and reliable way. As may be expected, the question of uniqueness is somewhat more complicated than in the single target case, as will be discussed next.

3. STRUCTURAL ISSUES

3.1 Uniqueness

The identification of linear systems from input/output data requires proper parameterization of their transfer function (or state-space representation). In the single-input single-output case all that is needed is a determination of the orders of the numerator and denominator polynomials. In the multivariable case more structual information is needed due to the fact that the numerator and denominator polynomial matrices can be transformed in various nontrivial ways without changing the order of the realization. To see this, let $U(z)$ denote a unimodular polynomial matrix [14]; then $H(z)=N(z)\,D(z)^{-1}=(N(z)U(z))(D(z)U(z))^{-1}$. In other words, $\tilde{N}(z) = N(z)U(z)$ and $\tilde{D}(z) = D(z)U(z)$ provide a matrix fraction description (MFD) of the transfer function $H(z)$, of the same order as $N(z),D(z)$. To obtain a unique parameterization of the transfer function its matrix fraction description $\{N(z), D(z)\}$ has to be put in a canonical form. Some of the commonly used forms are the Smith-McMillan form and the Popov or Polynomial-Echelon form. It is interesting to note that the model associated with the multitarget tracking problem (equations (12) (13)) has a diagonal denominator matrix. If $N(z),D(z)$ are irreducible, this implies uniquesness of the MFD. [To see this consider the class of unimodular matrices $U(z)$ such that $\text{diag}\{d_i(z)\}\cdot U(z)$ = diagonal. Only $U(z) = \text{diag}\,\{C_i\}$ for some constants C_i will satisfy this equation. If we further require that the leading coefficients of polynomials are unity, we must have $U(z) = I$]. The assumption of irreducibility of $N(z)$ and $D(z)$ will be generally satisfied, since it is unlikely that the delay structure $N(z)$ will bear any relationship to the target signals $D(z)$. Thus, if the diagonal structure of $D(z)$ is taken into account in the identification process, and if the orders of $D(z)$, $N(z)$ are known, a unique parameterization of the transfer function will be achieved.

For the tracking problem the uniqueness of $N(z)$, $D(z)$ is not too important as long as the transfer function $H(z)$ can be uniquely estimated. It is possible, in fact, to extract the information needed for tracking directly from the impulse response of the system $H(z)=N(z)D(z)^{-1}$, <u>without</u> looking at the numerator and denominator polynomials. By applying an impulse to the input u_1 of the first channel (see Figure 3) we will obtain at the outputs the impulse response of $1/d_1(z)$ delayed by various amounts. The spectrum of target No. 1 can be evaluated by computing the power spectrum of the impulse response, and its TDOA by cross-correlating the two outputs. A similar procedure can be carried out for Target No. 2. The point is that once the transfer function from u to y is known, we can effectively separate the different targets!

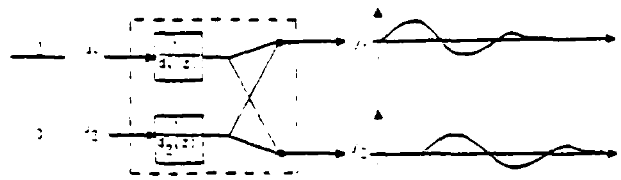


Figure 3. Using the Model to Obtain Information About Target No. 1.

The main difficulty in the tracking problem is that we have to estimate $H(z)$ from output measurements only, i.e., we know $y(t)$ but not $u(t)$. It is well-known that for stationary gaussian inputs $u(t)$ the transfer function $H(z)$ can be determined only up to a spectral equivalence class. In other words, all we can say is that $H(z)\,H(z^{-1})^T = S_y(z)$, the spectrum of the data $y(t)$. Making the usual assumption that both the poles and zeroes of $H(z)$ are within the unit circle (i.e., a stable, minimum phase plant), we are still left with nonuniqueness of the spectral factor due to the existence of unit spectral matrices. Let $S(z)$ denote a polynomial matrix such that $S(z)\,S(z^{-1})^T = I$. Then $H(z)$ and $H(z)S(z)$ have the same spectrum. To see that these exist nontrival unit-spectral matrices let Q denote an orthogonal matrix, i.e., $QQ^T = I$, and let $R(z)$ donte a diagonal delay matrix. i.e., $R(z) = \text{diag}\,\{z^{-k_i}\}$ for some integers k_i. It is easy to see that arbitrary products of matrices of these two types are a unit spectral factor. Consider for example $S(z)=QR(z)$; then $S(z)\,S^T(z)=QR(z)\,R(z^{-1})Q^T=QIQ^T=I$. The presence of unit spectral factors of the $R(z)$ type has the intuitive meaning as in the scalar case: the absolute propagation delays can not be determined from measurements of $y(t)$ alone. Only time differences can be determined.

In order to be able to separate the information related to the different targets it is essential to be able to get rid of the nonuniqueness introduced by the unit spectral factor. To do this we must use the *special structure of the denominator matrix* $D(z)$ and assume that $N(z),D(z)$ are irreducible (i.e., a minimal realization). First note that $N(z)D(z)^{-1}S(z)$ will, in general, have a larger order than $N(z)D(z)^{-1}$ unless $\det S(z)=1$. Thus, for the proper choice of model order we must have $S(z)=Q$. In other words, the only nonuniqueness left if we restrict our attention to

minimal order spectral factors of $S_y(z)$, is generated by orthogonal transformations. Next note that $H(z) = N(z)D(z)^{-1}Q = N(z)U(z)(Q^T D(z)U(z))^{-1}$, where $U(z)$ is a unimodular matrix. Since we know that $D(z)$ is diagonal, the nonuniqueness issue reduces to the question of what class of matrices $U(z)$ and Q will obey $Q^T D(z) U(z) =$ diagonal. By the uniqueness of the Smith-McMillan form [14] it follows that the only possible modifications of $D(z)$ are different orderings of the polynomials $\{d_1(z), \ldots, d_p(z)\}$. In other words $U(z) = Q$ where Q is a permutation matix (it may also include arbitrary sign changes). This fact has the following intuitive interpretation: since the initial labeling of the targets is arbitrary, we can only expect to define $H(z)$ up to column permutations. However, once a given labeling was determined, $H(z)$ will be fixed, and its MFD will be uniquely determined.

In summary: using the diagonal structure of $D(z)$ and assuming knowledge of the correct (minimal) order of the transfer function from $u(t)$ to $y(t)$, it is possible to obtain a unique (up to input labeling) spectral factor of $S_y(z) = H(z)\ H(z^{-1})$. The impulse response of the spectral factor $H(z)$ will provide the desired tracking parameters (TDOA's, spectra).

It should be noted that in addition to the structure of $D(z)$, we may use the special form of $N(z)$ which is known to be a delay matrix. This information can be used to relax the assumption of minimality and to obtain uniqueness for more general problem formulations (see, e.g., Section 5).

3.2 Right and Left Matrix Fraction Descriptions

The remaining question is one of parameter estimation. How to find a set of coefficients $N(z)$, $D(z)$ which will best fit (in the least-squares or maximum likelihood sense) a given time-series $\{y(t)\}$. Various techniques have been proposed in the literature for estimating the parameters of ARMA processes of the form

$$y(t) = -\sum_{i=1}^{p} A_i\ y(t-i) + \sum_{i=0}^{m} B_i\ u(t-i) \quad . \tag{15}$$

Most of the work in this area seems to have been done in the context of system identification (i.e., the known input case) [15]-[20], but some results are available on time-series modeling (unknown input case) [21]. Unfortunately, most of these techniques are not applicable to the estimation of $N(z)$, $D(z)$. Note that the ARMA model in equation (15) is naturally related to the Left Matrix Fraction Description (LMFD) of the transfer function from $u(t)$ to $y(t)$, while $N(z)$, $D(z)$ are its Right Matrix Fraction Description (RMFD). To see this we rewrite (15) as

$$A(z)\ Y(z) = B(z)\ U(z) \tag{16}$$

where

$$A(z) = I + A_1 z^{-1} + \ldots + A_p z^{-p},$$

$$B(z) = B_o + B_1 z^{-1} + \ldots + B_m z^{-m}$$

thus,

$$H(z) = A(z)^{-1}\ B(z) = N(z)\ D(z)^{-1}\ . \tag{17}$$

One way of estimating $N(z)$, $D(z)$ is to first estimate $A(z)$, $B(z)$ using any of the techniques described in [15]-[21] (see also Section 4), and then compute the RMFD of the transfer function $A(z)^{-1}B(z)$. The RMFD and LMFD are related by $A(z)N(z) - B(z)D(z) = 0$. Writing this equation in terms of the coefficients of A,B,N and D leads to the following matrix equation (written for simplicity for the case m=p=2):

$$\left[\begin{array}{ccc|ccc} A_o & & & B_o & & \\ A_1 & A_o & & B_1 & B_o & \\ A_2 & A_1 & A_o & B_2 & B_1 & B_o \\ & A_2 & A_1 & & B_2 & B_1 \\ & & A_2 & & & B_2 \end{array}\right] \left[\begin{array}{c} N_o \\ N_1 \\ N_2 \\ \hline D_o \\ D_1 \\ D_2 \end{array}\right] = 0 \tag{18}$$

where $A_o = I$. Note that if we fix the leading coefficient of $D(z)$, say $D_o=I$, equation (18) can be rewritten as a matrix equation with number of unknowns equal to the number of equations, and can therefore be solved for $\{N_i, D_i\}$. This matrix equation involved a (modified)Sylvester matrix with a very special structure. Using this structure Kung et al. [22]-[24] developed efficient algorithms for going from LMFD to RMFD and vice-versa. These algorithms utilize orthogonal transformations and are claimed to have good numerical properties. The problem of changing right to left matrix fraction descriptions is closely related to finding the greatest common divisor of polynomial matrices and to computing a minimal basis for the space spanned by such matrices [14].

A preferable solution would be to estimate directly the parameters $\{N_i, D_i\}$ of the RMFD. This can be done, for example, by using a maximum likelihood or prediction error formulation which leads to a nonlinear optimization problem. Such a solution will in general be computationally expensive. Some preliminary investigation seems to indicate the possibility of obtaining estimation algorithms for $N(z), D(z)$ which are of a comparable level of complexity to algorithms for estimating $A(z), B(z)$. However, this investigation is not yet complete.

It should be recalled that the desired target *information can be obtained directly* from the impulse response of the system, which can be computed <u>either</u> from the LMFD $(H(z)=A(z)^{-1}B(z))$ <u>or</u> from the RMFD $H(z)=N(z)D(z)^{-1}$ as indicated in Section 3.1. The main reason for wanting to estimate the RMFD is that it displays clearly the special structure of the system which is needed to force a unique solution of the spectral factorization problem. If the special structure of $N(z), D(z)$ could be mapped into an easily specified special structure for $A(z), B(z)$, we could work directly in terms of the LMFD. Unfortunately, the structure of $A(z), B(z)$ induced by $D(z)$ being diagonal and $N(z)$ being a delay matrix, seems to be complicated and difficult to use. It should also be noted that it is desirable to enforce the special structure already in the estimation process. This can be done easily in the RMFD, but not in the LMFD, which makes it even more important to develop a direct estimation technique for $N(z), D(z)$.

In summary, several approaches to the estimation of the model parameters were discussed: (i) Direct estimation of the RMFD, with $D(z)$ forced to be diagonal, (ii) Estimation of the LMFD using available ARMA modeling techniques. Transformation of the estimated LMFD into RMFD using available techniques. Checking the structure of $D(z)$ and finding the transformation needed to make it diagonal. This transformation will pin down a unique spectral factor. (iii) Estimate $A(z), B(z)$ with their form forced to have the particular structure induced by $N(z), D(z)$. Then compute the impulse response of $H(z) = A^{-1}(z)B(z)$ and from it the TDOA's and target spectra. Of these approaches only the second is fully developed at the current time, as described in Section 4.

3.3 Innovations Representation

In the discussion so far we have not mentioned explicitly the number of targets or sensors. Let us assume that there are N_T targets and N_S sensors and that $N_T \leq N_S$. Consider the spectrum of $y(t)$ given by equation (14):

$$S_y(z) = N(z)D(z)^{-1} D(z^{-1})^{-T} N(z^{-1})^T + \Sigma_v \qquad (19)$$

where Σ_v denotes the covariance matrix of the measurement noise $v(t)$. Note that if $\Sigma_v=0$ and $N_T<N_S$, $S_y(z)$ will not be a full rank matrix! (An equivalent way of stating this fact is to say that the innovations representation of $y(t)$ will be singular, and some of the components of the innovations vector $\varepsilon(t)$ will be linearly dependent). In fact, the rank of the covariance matrix $S_y(z)$ (or the innovations covariance matrix R^{ε}) will be equal to the number of targets, as can be seen from

$$S_y(z) = \underbrace{N(z)D(z)^{-1}}_{N_S \times N_T} \underbrace{D(z^{-1})^{-T} N(z^{-1})^T}_{N_T \times N_S} \qquad (20)$$

Thus, the number of targets can be estimated by testing the rank of $S_y(z)$ (R^{ε}). This test will be, of course, sensitive since any measurement noise will make $\Sigma_v>0$ and cause $S_y(z)$ (R^{ε}) to become full rank. However, by looking at the relative magnitudes of the eigenvalues of the covariance matrix, we can obtain a more robust estimate of the number of targets. This procedure is described in more detail in Section 4. In general, we will work with a "square" transfer function, i.e., $N(z)$ and $D(z)$ (or $A(z)$ and $B(z)$) are $N_S x N_S$ matrices, and decide on the actual number of targets after performing the model fitting. If $N_T<N_S$, we may think of this as having N_T actual targets and N_S-N_T fictitious targets which generate no signals (i.e., the corresponding inputs $u_i(t)=0$).

3.4 Beamforming Interpretation

A standard approach to passive tracking is beamforming. In its simplest form, beamforming consists of forming a sum of delayed sensor outputs, to get a scalar signal $f(t)$ which then undergoes further processing. In the single target case, this can be viewed as forming an inner product of the data vector $Y(z)$ with a "steering" vector $W(z)$,

$$F(z) = W(z)^T Y(z) = W(z)^T (N(z) X(z) + V(z))$$

The steering vector is chosen so that it "removes" the numerator polynomial which represents the propagation model, i.e., choose $W(z)$ so that

$$W(z)^T N(z) = z^{-T}$$

and then the received signal $f(t)$ is just a delayed version of the target signal $x(t)$. To see this more clearly, consider $N(z)$ as given in equation (6), and let

$$W(z)^T = [\alpha z^{-T_2} \; z^{-T_1}]/2\alpha \quad .$$

Then clearly $W(z)^T N(z) = z^{-(T_1+T_2)}$. In the multi-target case $X(z)$ will be a matrix which is chosen so that $W(z)^T N(z) = \mathrm{diag}\,\{z^{-T_i}\}$. Thus, the operation of beamforming can be thought of as finding the inverse of $N(z)$ in a generalized sense. Note that beamforming is very closely realted to the RMFD representation. In fact, the whitening filter $U(z)=D(z)N(z^{-1})Y(z)$ can be considered as a combination of beamforming $F(z)=N(z)^{-1}Y(z)$ and spectral filtering $U(z)=D(z)F(z)$, performed simultaneously for all targets!

4. THE ALGORITHM

A candidate algorithm for estimating the parameters A_i, B_i is described below. This algorithm is the multivariable version of the Recursive Maximum Likelihood (RML) techique, described in [25] for the single input/output case.

Let us rewrite the ARMA model of equation (15) as

$$y_t = \theta^T \phi_t + \varepsilon_t \qquad (21)$$

where

$$\theta^T = [A_1, \ldots, A_p, B_o, \ldots, B_m],$$

an $N_S x(p+m+1)N_S$ matrix

$$\phi_t^T = [-y_{t-1}^T, \ldots, -y_{t-p}^T, \varepsilon_{t-1}^T, \ldots, \varepsilon_{t-m-1}^T]$$

an $1 \times (p+m+1)N_S$ vector

or it can be decoupled into N_S equations of the type

$$y_t^j = \phi_t^T \theta^j + \varepsilon_t^j \quad , \quad j=1, \ldots, N_S \qquad (22)$$

where θ^j is the j-th column of θ.

The estimation algorithm for θ is given by

$$\hat{\theta}_{t+1}^j = \hat{\theta}_t^j + K_{t+1} \varepsilon_{t+1}^j, \quad j=1, \ldots, N_S \qquad (23)$$

where K_{t+1} is a gain vector (common to all of the θ^j's), and

$$\varepsilon_{t+1}^j = y_{t+1}^j - \phi_{t+1}^T \hat{\theta}_t^j, \quad j=1, \ldots, N_S \qquad (24)$$

These equations can be written in a more compact form as

$$\hat{\theta}_{t+1} = \hat{\theta}_t + K_{t+1} \varepsilon_{t+1}^T \qquad (25a)$$

$$\varepsilon_{t+1} = y_{t+1} - \hat{\theta}_t^T \phi_{t+1} \qquad (25b)$$

The gain vector K can be computed by

$$K_{t+1} = P_t \tilde{\phi}_{t+1} / (\lambda_t + \tilde{\phi}_{t+1}^T P_t \tilde{\phi}_{t+1}) = P_{t+1} \tilde{\phi}_{t+1} \qquad (26a)$$

$$P_{t+1} = [P_t - P_t \tilde{\phi}_{t+1} \tilde{\phi}_{t+1}^T P_t / \lambda_t + \tilde{\phi}_{t+1}^T P_t \tilde{\phi}_{t+1})] / \lambda_t \qquad (26b)$$

where λ_t is an exponential weighting factor, and $\tilde{\phi}_t$ is a filtered version of the ϕ vector, i.e., y, ε in the ϕ vectors are replaced by pre-filtered versions $\tilde{y}$ and $\tilde{\varepsilon}$ where

$$\tilde{y}_t = D^{-1}(z)\, y_t \quad , \quad \tilde{\varepsilon}_t = D^{-1}(z)\, \varepsilon_t, \qquad (27a)$$

$$D(z) = I + D_1 z^{-1} + \ldots + D_k z^{-k} \qquad (27b)$$

The pre-filter $D(z)$ is typically chosen as $D(z) = \hat{B}(z)$. However, we found that improved convergence can sometimes be achieved by using $D(z)=\hat{A}(z/\alpha)$, $\alpha \leq 1$. See [13] for more details.

The innovations sequence ε_t produced by the RML will generally have correlated entries (i.e., a non-diagonal innovations matrix). Since we assume the target signals to be uncorrelated, it is necessary to work with the normlaized innovaciona sequence $\bar{\varepsilon}_t$

$$\bar{\varepsilon}_t = R^{-\varepsilon/2} \varepsilon_t \qquad (28)$$

where $R_t^{\varepsilon/2}$ is the (lower triangular) square root of the innovation covariance. R_t^{ε} can be computed recursively by

$$R_{t+1}^{\varepsilon} = (1-w_t)\, R_t^{\varepsilon} + w_t\, \varepsilon_t\, \varepsilon_t^T \tag{29a}$$

$$w_{t+1} = w_t/(w_t + \lambda_t) \tag{29b}$$

or in square root form

$$\begin{bmatrix} \sqrt{1-w_t}\, R_t^{\varepsilon/2} \\ \sqrt{w_t}\;\; \varepsilon_t^T \end{bmatrix} \xrightarrow{\text{orthogonal transformation}} \begin{bmatrix} R_{t+1}^{\varepsilon/2} \\ 0 \end{bmatrix} \tag{30}$$

The normalized innovations will be computed by solving

$$R_t^{\varepsilon/2}\, \tilde{\varepsilon}_t = \varepsilon_t \tag{31}$$

which can be done by back substitution since $R^{/2}$ is upper triangular. Special care has to be taken, since R^{ε} may be singular, i.e., some of the diagonal elements of $R^{\varepsilon/2}$ may be zero (or very small). In that case, the corresponding elements of ε_t will be set to zero. This can be done easily in the back substitution routine. The number of diagonal elements of $R^{\varepsilon/2}$ which are greater than some threshold value, will indicate the number of targets, as was explained earlier.

To estimate TDOA's and target spectra, the impulse response of $H(z)=\hat{A}(z)^{-1}\hat{B}(z)\, R^{\varepsilon/2}$ will be computed for each input/output pair. By cross-correlating the impulse responses obtained for a given input i, the TDOA's corresponding to target number i will be obtained. The power spectrum of the impulse response will provide an estimate of the target spectrum. There is, however, one remaining problem: the transfer function $H(z)$ may "scramble up" the targets by an orthogonal transformation. In other words, the correct transfer function will generally be $H(z)Q$, where Q is an orthogonal transformation that needs to be determined (alternatively: find the correct way of taking the matrix square-root $R^{\varepsilon/2}$).

In the case where N_S is small it is possible to take a general orthogonal transformation matrix and look at the cross-correlations of the impulse reponses. obtained for different rotations. If the wrong rotation is used, the cross-correlation function will have multiple peaks corresponding to several targets. If the correct orthogonal transformation is used only a single peak will occur corresponding to a single target. This is somewhat similar to beam steering (except that it is done in a different space).

A more systematic way for determining the orthogonal transformation Q is to compute the RMFD of $\hat{A}(z)^{-1}\hat{B}(z)R^{\varepsilon/2}$. One technique for doing this is presented in [22] and will not be repeated here. Once the RMFD $\hat{N}(z),\hat{D}(z)$ is calculated, Q can be determined as the transofrmation needed to make $\hat{D}(z)$ diagonal (or as nearly diagonal as possible).

Finally, we should note that the algorithm presented here was chosen *mainly for convenience of implementation*. Many other algorithms can be used to estimate A_i, B_i and it is not clear at this time which one is best for multitarget tracking applications. However, the RML technique seems adequate for testing the proposed modeling approach. Some preliminary simulation results are given in the Appendix.

5. SOME EXTENSIONS

The multitarget tracking problem was formulated in Section 2 for fixed targets with AR spectra, and a simple propagation model. In this section we briefly describe how the ARMA modeling approach can be extended to other classes of tracking problems.

5.1 Moving Targets

When targets and sensor locations are fixed in space, the model presented in Section 2 will be time-invariant. (This follows from the assumptions of the stationary target signals and a time-invariant propagation medium). Relative target motion will have two important effects: (i) The system relating $y(t)$ to $x(t)$ will be time varying and (ii) the signals generated by the target will experience doppler shifts. The first effect means that we now have to identify an ARMA model with time-varying coefficients. (In fact, only the N_i coefficients are time varying). This is a difficult task even in the single channel case, although it has been done successfully in various applications such as speech processing. In many situations (e.g., sonar systems) target motion is sufficiently slow compared to the sampling rate, so that the parameter estimation algorithm will be able to track parameter changes. More precisely, as long as the target/sensor geometry can be considered fixed over the time interval needed to get a reasonable location estimate (integration time) techniques which work well for fixed targets will still be expected to work. If the geometry changes sufficiently fast, a time-varying problem formulation is unavoidable. For example, we may try to parameterize the coefficients N_i of the delay matrix by $N_i(t) = N_{i,o}+N_{i,1}t+N_{i,2}t^2$. The coefficients $N_{i,o}, N_{i,1}, N_{i,2}$ can be related to the target location and its velocity.

In many applications, the signals received from moving targets can undergo significant doppler shifts, even if the target/sensor geometry is only slowly time-varying. *These frequency shifts provide information* about target velocity and about its location (e.g., by looking at differences of doppler shifts between pairs of sensors). Thus, it is important to see how doppler effects fit into the modeling framework. Consider an AR process $x(t)$ generated by a target, where $x(t)=(1/d(z))\,u(t)$. For narrowband processes, it can be shown that the doppler shifted signal will still be an AR process, except that the AR coefficients d_i are changed. In fact, if $d(z)=(z-z_1)\ldots(z-z_n)$ then its doppler shifted version will be $d(z)=(z-z_1^{\alpha})\ldots(z-z_n^{\alpha})$ where the parameter α is determined by target velocity and propagation velocity. Thus, the model for the observed signal will have the form

$$y(z) = H(z)\, U(z) + V(z) \tag{32a}$$

$$H(z) = [z^{-T_{ij}}/d_{ij}(z)],\quad N_S \times N_T \text{ matrix} \tag{32b}$$

where $d_{ij}(z)$ is the AR representation of the signal generated by target i, as seen by sensor j, and T_{ij} is the delay from target i to sensor j. Note that the parameter of $d_{ij}(z)$, $j=1, \ldots, N_S$ are not independent, but are related in a special way. An examination of these parameters provides not only a spectral estimate, but also information about target velocity and location. The structure of this model has many interesting features which are still under investigation. Of particular interest is the question of how to obtain a unique spectral factor, in view of the fact that $D(z)$ is no longer diagonal. As may be expected, uniqueness can be achieved under fairly mild conditions. Estimating the parameters of such models from noisy data, in a way that takes into account their special structure is not yet fully developed.

5.2 ARMA Sources

While AR models provide an adequate signal representation in a large class of practical cases, it is sometimes necessary to consider the more general ARMA representation. This is particularly true for wideband processes or narrowband processes in additive noise. The signal model will have in this case the form

$$X(z) = \bar{N}(z)\,\bar{D}(z)^{-1}\,U(z) \tag{33}$$

where $\bar{N}(z)$, $\bar{D}(z)$ are both diagonal,

and

$$Y(z) = (N(z)\,\bar{N}(z))\,\bar{D}(z)^{-1}\,U(z) + V(z). \tag{34}$$

Note that the numerator of the RMFD is now a mixture of the delay matrix and the spectral zeroes of the targets. However, $\bar{D}(z)$ is still diagonal and therefore most of the discussion in Section 3 still holds true. The TDOA's and target spectra can still be estimated from the impulse response of the estimated model. Because of the diagonal structure of $\bar{N}(z)$ it is theoretically possible to separate $N(z)$ and $\bar{N}(z)$ by looking for common factors in the columns of $N(z)\,\bar{N}(z)$. *In summary, having an ARMA spectral* model does not complicate significantly the modeling approach.

5.3 ARMA Propagation Models

The simplest form of multipath propagation is caused by a signal being reflected by some object which lies outside the direct line of propagation, causing an extraneous delayed version of the signal to arrive at the sensors. This type of multipath propagation can be represented by a MA model of the type used in earlier sections. Often, a more complicated type of propagation occurs. The signal can undergo multiple reflections of the type encountered, for instance, when a sound wave propagates in a room (e.g., the "cocktail party" problem: locating people who are speaking inside a room). The sound is reflected from one wall, bounces off the opposite wall and again from the first wall. Similar effects occur when seismic waves propagate in the earth or when sound propagates in the ocean (reflected from the water/air and water/ocean floor interfaces). This phenomena is best modeled by an AR type model.

In general, a realistic propagation model will combine both types of multipath propagation and will therefore be an ARMA model, i.e.,

$$\begin{aligned} Y(z) &= N(z)\,D(z)^{-1}\,X(z) + V(z) = \\ &= N(z)\,[\bar{D}(z)\,D(z)]^{-1}\,U(z) + V(z) \end{aligned} \tag{35}$$

where $N(z)$, $D(z)$ are the propagation model and $\bar{D}(z)$ is the signal model.

The structure of the propagation model depends, of course, on the particular application being considered and on the physical properties of the propagation medium. In its simplest form $D(z)$ will have a diagonal form, meaning that the propagation from a given target to all sensors has the same kind of propagation effects, except for different delays which are represented in $N(z)$. In general, more complicated forms of propagation are possible in which case $D(z)$ will be non-diagonal.

When $D(z)$ is diagonal, the structure of the transfer function $H(z) = N(z)\,(\bar{D}(z)\,D(z))^{-1}$ is similar to the pure delay case. The only problem is that the signal spectrum is mixed up with the poles of the propagation medium. Under certain situations it may be possible to separate the two. Consider the situation where the time dependence of the signal spectrum and the propagation model are different. For example, consider a nonstationary source, like speech, which stays fixed in space so that the propagation model is time-invariant. By repeating the computation of $\bar{D}(z)\,D(z)$ over several time intervals and computing its roots, $D(z)$ will be found from those roots which do not change, while $\bar{D}(z)$ will correspond to the roots which are varying from one time interval to another. As another example, consider a moving target that causes the propagation model to change. Here $\bar{D}(z)$ will have fixed roots and $D(z)$ roots which vary over time. If either $D(z)$ or $\bar{D}(z)$ are nondiagonal the structure of the transfer problem becomes more complicated and has to be examined more carefully. This will not be done here.

5.4 Direct Estimation of Source Location

The set of TDOA's contain all the information required to locate the targets, i.e., to find their bearing/range. Location estimation is typically performed in two steps: First, estimate the TDOA's (Δ_{ij}) using the algorithms described earlier, or a more standard generalized cross-correlation technique. Then compute target locations from the set of Δ_{ij}'s using the approach presented in [12], or the hyperbolic line-of-position technique.

A different approach will be to try and estimate directly the target coordinates. Since the functional relationship between the target coordinates and the TDOA's is known, it is possible to re-formulate the problem as a coordinate estimation problem. Some preliminary studies seem to indicate the feasibility of the direct approach, but no conlcusive results are available at this time.

6. CONCLUSIONS

We introduced a framework which relates various problems that arise in multitarget tracking to the parameters and the structure of a related multivariable linear system. This approach motivates some interesting questions regarding the modeling of vector ARMA processes under various structural constraints. The results presented here are only preliminary and the objective was mainly to lay the groundwork for further research.

The proposed approach has several features which are important from a practical standpoint, including: treating multiple targets without performing a separate association problem (target assocation as well as line association), simultaneous estimation of TDOA and target spectra and possibility of handling multipath effects. However, considerable testing and performance *evaluation needs to be done before the usefulness of* the ARMA modeling approach can be fully realized.

ACKNOWLEDGMENTS

The author gratefully acknowledges the contribution of Professor M. Morf of Stanford University who first proposed the modeling approach and generously provided his ideas and guidance.

REFERENCES

1. E.C. Fraser and L. Meier, "Mathematical Models and Optimum Computation for Computer-Aided Active Sonar Systems," U.S. Navy Electronic Lab., SRI Final Rep. (First Year), San Diego, CA, Contract N123-(953)54486A, March 1967.
2. P. Smith and G. Buechler, "A Branching Algorithm for Discriminating and Tracking Multiple Objects," IEEE Trans. Auto. Control, Vol. AC-20, February 1975, pp. 101-104.
3. G.A. Ackerson and K.S. Fu, "On State Estimation in Switching Environments," IEEE Trans. Auto. Control, Vol. AC-15, Feb. 1970, pp. 10-17.
4. H. Akashi and H. Kumamoto, "Random Sampling Approach to State Estimation in Switching Environments," Automatica, Vol. 13, July 1977, pp. 429-434.
5. A.G. Jaffer and S.C. Gupta, "Recursive Bayesian Estimation with Uncertain Observation," IEEE Trans. Infor. Theory, Vol. IT-17, September 1971, pp. 614-616.
6. ______. "Optimal Sequential Estimation of Discrete Processes with Markov Interrupted Observations," IEEE Trans. Auto. Control, Vol. AC-16, October 1971, pp. 471-475.
7. N.E. Nahi, "Optimal Recursive Estimation with Uncertain Observations," IEEE Trans. Inform. Theory, Vol. IT-15, July 1969, pp. 456-462.
8. C.L. Morefield, "Application of 0-1 Integer Programming to Multitarget Tracking Problems," in Proc. IEEE Conf. Decision and Control, Dec. 1975 and IEEE Trans. Auto. Control, Vol. AC-22, June 1977, pp. 302-312.
9. R.A. Singer, R.G. Sea, and K. Housewright, "Derivation and Evaluation of Improved Tracking Filters for Use in Dense Multitarget Environments," IEEE Trans. Inform. Theory, Vol. IT-20, July 1974, pp. 423-432.
10. R.W. Sittler, "An Optimal Data Association Problem in Surveillance Theory," IEEE Trans. Mil. Electron, Vol. MIL-8, April 1964, pp. 125-139.
11. Y. Bar-Shalom, "Tracking Methods in Multitarget Environment," IEEE Trans. on Auto. Control, Vol. AC-23, No. 4, pp. 618-626, August 1978.
12. J.M. Delosme, M. Morf and B. Friedlander, "Estimating Source Location from Time Difference of Arrival: A Linear Equations Approach," Technical Report No. M355-1, SEL-79-010, Stanford University, 31 March 1979.
13. B. Friedlander, "Multitarget Tracking Studies: System Description and Preliminary Evaluation," Systems Control, Inc. Technical Report, TM 5334-04, June 1980.
14. T. Kailath, Linear Systems, Prentice-Hall, Inc. 1980.
15. A. Gauthier and I.D. Landau, "On the Recursive Identification of Multi-Input Multi-Output Systems," Automatica, Vol. 14, pp. 609-614, 1978.
16. B.D.O. Anderson, "An Approach to Multivariable System Identification," Automatica, Vol. 13, pp. 401-408, 1977.
17. R. Guidorzi, "Canonical Structures in the Identification of Multivariable Systems," Automatica, Vol. 11, pp. 361-374, 1975.
18. E. Tse and H.L. Weinert, "Structure Determination and Parameter Identification for Multivariable Stochastic Linear Systems," IEEE Trans. Automatic Control, Vol. AC-20, No. 5, pp. 603-613, October 1975.
19. L.C. Suen and R. Liu. "Determination of the Structure of Multivariable Stochastic Linear Systems," IEEE Trans. Automatic Control, Vol. AC-23, No. 3, pp. 458-464, June 1978.
20. B.W. Dickinson, M. Morf and T. Kailath, "A Minimal Realization Algorithm for Matrix Sequences," IEEE Trans. Automatic Control, Vol. AC-19, No. 1, February 1974, pp. 31-38.
21. A. Jakeman and P. Young, "Refined Instrumental Variable Methods of Recursive Time-Series Analysis, Part II: Multivariable Systems," Int. J. Control, Vol. 29, No. 4, pp. 621-644, 1979.
22. S. Kung, T. Kailath and M. Morf, "Fast and Stable Algorithms for Minimal Design Problems," Proc. of the Fourth IFAC Int. Symposium on Multivariable Technological Systems, Fredericton, Canada, 4-8 July 1977.
23. S. Kung, T. Kailath and M. Morf, "A Fast Projection Method for Canonical Minimal Realization," Proc. of the 1976 IEEE Conf. on Decision and Control.
24. S. Kung, "Optimal Hankel-Norm Model Reductions - Scalar Systems," Proc. of the Joint Automatic Control Conf., San Francisco, August 1980.
25. T. Soderstrom, L. Ljung and I. Gustavsson, "A Comparative Study of Recursive Identification Methods," Report 7427, Dept. Automatic Control, Lund, Sweden, 1974.

APPENDIX C

MTS PARAMETER ESTIMATION ALGORITHMS

A number of algorithms for estimating the parameters of AR, ARX and ARMA models were coded and tested during the MTS project. These algorithms can be divided into three classes: recursive parameter estimation algorithms of the RML type, non-recursive AR and ARMA algorithms and recursive lattice algorithms. In this appendix we list the algorithms that are currently available and either give a brief description or a reference to a more complete description.

C.1 RECURSIVE IDENTIFICATION ALGORITHMS (IDN1)

1. Recursive Least-Squares (RLS)

   This algorithm estimates the parameters of the ARX model

$$y_t = -\sum_{i=1}^{NA} a_i \, y_{t-1} + \sum_{i=1}^{NB} b_i \, u_{t-i} + v_t$$

   where u_t is a white noise process. The algorithm is given by

$$\theta = [a_1, \ldots, a_{NA}, b_1, \ldots, b_{NB}]$$

$$\phi_t = [-y_{t-1}, \ldots -y_{t-NA}, u_{t-1}, \ldots, u_{t-NB}]$$

$$P_t = [P_{t-1} - P_{t-1}\phi_t\phi_t^T P_{t-1}/(\lambda_t + \phi_t^T P_{t-1}\phi_t)]/\lambda_t$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} + P_t\phi_t\,(y_t - \phi_t^T\hat{\theta}_{t-1})$$

$$\lambda_t = \lambda\lambda_{t-1} + (1-\lambda)$$

$$\hat{\theta}_0 = 0, \; P_0 = \sigma I$$

2. Extended Least Squares (RML1)

This is the RML algorithm described by equations (2) - (7) with $D_t(z) = 1$ and with e_t replaced by $\varepsilon_t = y_t - \phi_t^T\hat{\theta}_{t-1}$.

3. Extended Least Squares (RMLP)

This is the RML algorithm described by equations (2) - (7) with $D_t(z) = 1$.

4. Recursive Maximum Likelihood (RML)

This is the RML algorithm described by equations (2) - (7).

5. Square-Root form of the RML (NORM)

This is the algorithm described in section 2.1 (iii).

6. Stochastic Approximation RLS (LMS)

This is a version of the Widrow-Hoff gradient research LMS algorithm.

$$\theta = [a_1, \ldots, a_{NA}]^T$$

$$\phi_t = [-y_{t-1}, \ldots, -y_{t-NA}]^T$$

$$\varepsilon_t = y_t - \phi_t^T\hat{\theta}t-1$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} + (\mu/R_t)\, \phi_t \varepsilon_t$$

$$R_t = \lambda R_{t-1} + (1-\lambda)\sum_{i=1}^{NA} y_{t-i-\Delta}^2 , \qquad R_0=1 , \quad 0<\Delta<10$$

C.2 NONRECURSIVE ALGORITHMS (STAN)

1. Maximum Entropy Method (MEM)

This is an implementation of Burg's technique for autoregressive spectral estimation. The code used was taken from [21].

2. Covariance Method (COV)

This is an implementation of the covariance method of speech analysis as given by [22, pp. 52]. The resulting AR coefficients are used to compute the spectrum.

3. Modified Yule-Walker Method (MYW)

This is an algorithm for estimating ARMA parameters. The algorithms estimates first the AR parameters, using the following modified Yule-Walker equations:

$$\begin{bmatrix} R_{NA} & & R_1 \\ R_{NA+1} & \cdots & R_2 \\ \vdots & & \vdots \\ R_{NR-1} & \cdots & R_{NR-NA} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_{NA} \end{bmatrix} = \begin{bmatrix} R_{NA+1} \\ \vdots \\ \vdots \\ R_{NR} \end{bmatrix}$$

where

$$R_i = \frac{1}{N} \sum_{t=i+1}^{N} y_t y_{t-i} \qquad 0 < i < NR$$

This overdetermined system of equations is solved by a least-squares algorithm involoving singular value decomposition (LLSQF in the IMSL).

The MA coefficients are computed next using the following equation

$$\begin{bmatrix} b_{-NA} \\ \vdots \\ b_0 \\ \vdots \\ b_{NA} \end{bmatrix} = \begin{bmatrix} R_0 & R_1 & & R_{2NA} \\ R_1 & \ddots & & \\ & \ddots & \ddots & R_1 \\ R_{2NA} & & R_1 & R_0 \end{bmatrix} \begin{bmatrix} a_{NA} & a_{NA} & \cdots & 0 \\ a_{NA-1} & \vdots & & \\ \vdots & a_1 & \cdots & a_{NA} \\ 1 & 1 & \ddots & a_{NA-1} \\ 0 & & & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_{NA} \end{bmatrix}$$

The spectrum is finally computed by

$$S(z) = \frac{b_{-NA} z^{NA} + \ldots b_{-1} z + b_0 + b_1 z^{-1} + \ldots + b_{NA} z^{-NA}}{(1 + a_1 z^{-1} + \ldots + a_{NA} z^{-NA})\,(1 + a_1 z + \ldots + a_{NA} z^{NA})}$$

where $z = \exp(-j\omega)$

C3. RECURSIVE LATTICE ALGORITHMS (LADD)

1. The Normalized AR Lattice (ARN)
   This algorithm implements the normalized algorithm described in [23]. The AR coefficients are computed by Levinson's algorithm rather than by the true whitening lattice. See [24] for details.

2. The Unnormalized AR Lattice (ARU)
   This is the normalized lattice with its output properly scaled to give the unnormalized prediction error. See [24] for details.

3. The Two Channel Lattice (KNWN)
   This is the ARMA lattice for the known input case [25].

4. The ARMA Lattice (ARMA)
   This is the ARMA lattice for the *unknown input* case with the prediction error fed back to the input [25].

5. The AML Lattice (AML)
   This is the ARMA lattice for the unknown input case with the residual fed back to the input [25].

6. The Normalized Lattice (ARL)
   This is the normalized AR lattice with the true whitening filter for computing AR coefficients [26].

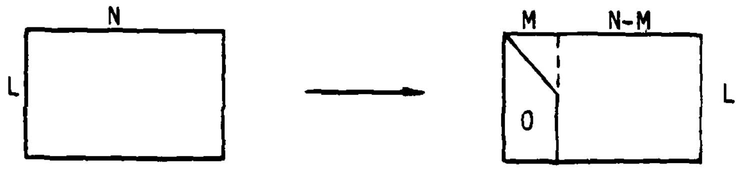
7. The Joint Process Lattice (ARJ)
   This is the normalized joint process AR lattice. The AR coefficients are computed by a true whitening filter [26].

8. The Sliding Window Lattice (ARS)
   This is the sliding window AR lattice form [26].

APPENDIX D

TRIANGULARIZATION BY HOUSEHOLDER TRANSFORMATION

The following routine will take an LxN matrix and upper-triangularize its first M columns.



Note: M < min {N,L}

Input: an LxN matrix S(I,J)
M = # of columns to be triangularized

Output: an LxN (partially) upper triangular matrix S(I,J)

```
   DO 4 J=1, M
    σ = Z     @ Z = zero
   DO 1 I=J, L
   v(I) = S(I,J)                              @ v(N) is a work vector
   S(I,J) = Z
1   σ = σ + v(I) ** 2
    IF (σ < Z) GO TO 4
```

Comment: If σ=Z, column J is zero and this step of the reduction is omitted.

```
    σ = SQRT (σ)
    IF (v(J)<Z) σ = -σ
    S(J,J) = σ
```

```
      v(J) = v(J) - σ
      σ = ONE/(σ * v(J))                                    @ ONE = 1.
     DO 3 K=J+1, N
      α = Z
     DO 2 I=J, L
 2    α = α + S(I,K) * v(I)
      α = α * σ
     DO 3 I=J, L
 3   S(I,K) = S(I,K) +  σ * v(I)
 4   CONTINUE
```

In the system identification problem, we want full triangularization of a square matrix, thus L=M=N.

Distribution List for
"Multitarget Tracking Studies Phase II Report"
All Addressees Receive One Copy Unless Otherwise Specified

	Copies
Naval Coastal Systems Center Code 741 Attn: Mr. C. M. Bennett Panama City, FL 32401	1
Naval Electronic Systems Command (NELEX 320) Attn: John Schuster National Center No. 1 Arlington, VA 20360	1
Dr. M.J. Fischer Defense Communications Agency Defence Communications Engineering Center 1860 Wiehle Avenue Reston, VA 22090	1
Professor Harold A. Titus Department of Electrical Engineering Naval Postgraduate School Monterey, CA 93940	1
Mr. Dan Leonard Code 8105 Naval Ocean Systems Center San Diego, CA 92152	1
Dr. Alan F. Petty Code 7930 Naval Research Laboratory Washington, D.C. 20375	1
Professor P.A.W. Lewis Department of Operations Research Naval Postgraduate School Monterey, CA 93940	1
Professor D.P. Gaver Department of Operations Research Naval Postgraduate School Monterey, CA 93940	1
Professor G.S. Watson Department of Statistics Princeton University Princeton, NJ 08540	1
Professor M. R. Leadbetter Department of Statistics University of North Carolina Chapel Hill, NC	1
Professor T. W. Anderson Department of Statistics Stanford University Stanford, CA 94305	1
Professor M. Rosenblatt Department of Mathematics University of California, San Diego La Jolla, CA 92093	1
Professor E. Parzen Institute of Statistics Texas A&M University College Station, TX 77840	1
Professor H.L. Gray Department of Statistics Southern Methodist University Dallas, TX 75275	1
Professor E. Masry Department of Applied Physics and Information Science University of California La Jolla, CA 92093	1
Professor L.L. Scharf, Jr. Department of Electrical Engineering Colorado State University Fort Collins, CO 80521	1
Professor M.J. Hinich Department of Economics Virginia Polytechnic Institute and State University Blacksburg, VA 24061	1
J. S. Lee Associates, Inc. 2001 Jefferson Davis Highway Suite 802 Arlington, VA 22202	1

Copies	

Professor Bernard Widrow 1
Stanford Electronics Laboratories
Stanford University
Stanford, CA 94305

Professor Grace Wahba
Department of Statistics
University of Wisconsin
Madison, Wisconsin 53706 1

Professor S.C. Schwartz
Department of Electrical Engineering
and Computer Science
Princeton University
Princeton, NJ 08540 1

Professor Charles R. Baker
Department of Statistics
University of North Carolina
Chapel Hill, NC 27514 1

Professor Balram S. Rajput
Department of Mathematics
University of Tennessee
Knoxville, TN 37916 1

Professor C. H. Chen
Department of Electrical Engineering
Southeastern Massachusetts University
North Dartmouth, MA 02747 1

Professor K. S. Fu
School of Electrical Engineering
Purdue University
West Lafayette, Indiana 47907 1

Professor Melvin F. Janowitz
Department of Mathematics and
Statistics
University of Massachusetts
Amherst, MA 01002 1

Professor M. M. Rao
Department of Mathematics
University of California
Riverside, CA 92521 1

Professor James A. Cadzow
Department of Electrical & Computer
Engineering
Arizona State University
Tempe, AZ 85287 1

Copies

Professor Stanley L. Sclove
Department of Mathematics
University of Illinois at
Chicago Circle
Box 4348
Chicago, IL 60680 1

Professor Solomon W. Golomb
Electronic Sciences Laboratory
University of Southern Calfornia
Los Angeles, California 90007 1

Professor Charles B. Bell
Department of Biostatistics
University of Washington
Seattle, Washington 98195 1

Professor Jerry D. Gibson
Department of Electrical Engineering
Texas A&M Research Foundation
College Station, Texas 77843 1

Office of Naval Research, Branch Office
Western Regional Office
1020 East Green Street
Pasadena, CA 91106 1

Professor Marshall E. Frazer
Applied Research Laboratories
University of Texas
P.O. Box 8029
Austin, Texas 78712 1

Professor Donald W. Tufts
Department of Electrical Engineering
University of Rhode Island
Kingston, Rhode Island 02881 1

Dr. C. Carter
Naval Underwater Systems Center
New London Laboratory
Code 313
New London, CT 06320 2

Naval Underwater Systems Center
Code 351
Code 352
Newport, RI 02840

Naval Sea Systems Command
Washington, D.C. 20360
Code 63R-1
Code 63R-16
Code 63R-13

	Copies
Mr. J. Conrad Naval Intelligence Support Center Code 20 Suitland, MD 20390	1
Naval Research Laboratory Washington, DC 20375 Code 2627 Code 5308 Code 7932	2
Dr. Byron D. Tapley The University of Texas At Austin Department of Aerospace Engineering and Engineering Mechanics Austin, TX 78712	1
Defense Documentation Center Cameron Station Alexandria, VA 22314	12
Office of Naval Research Code 411 SP Code 220 Arlington, VA 22217	2
Dr. Fred W. Weidmann Tracor, Inc. Tracor Science & Systems 6500 Tracor Lane Austin, TX 78721	1
Naval Ocean Systems Center Code 6212 Code 632 San Diego, CA 92152	2
Naval Surface Weapons Center White Oak Laboratory Code U-20 Silver Spring, MD 20910	2
Dr. Yaakov Bar-Shalom The University of Connecticut Department of Electrical Engineering and Computer Science Box U-157 Storrs, CT 06268	

	Copies
Manager, ASW Systems Project Office Naval Material Command ASW-118 Washington, DC 20360	1
Dr. Thomas E. Fortmann Bolt Beranek and Newman Inc. 50 Moulton Street Cambridge, MA 02138	1